APPROACHES TO DEMAND FORECASTING AND CAPACITY EXPANSION PLANNING FOR ELECTRICITY

By

RABINDRANATH CHOUDHURY

TH
621·319
C457a

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

AUGUST, 1976/APRIL, 1991

APPROACHES TO DEMAND FORECASTING AND CAPACITY EXPANSION PLANNING FOR ELECTRICITY

A Thesis Submitted
in
Partial Fulfillment of the
Requirements for the Degree of

MASTER OF TECHNOLOGY

BY

Rabindranath Choudhury

to the

Department of Mechanical Engineering
Indian Institute of Technology
Kanpur - 208016

August 1976

ENTR... RY
Acc. No. A112190
TH
621.319
C457a
E-1976-M-CHO-APP

CERTIFICATE

5/4/91

It is certified that the work contained in the thesis entitled "APPROACHES TO DEMAND FORECASTING AND CAPACITY EXPANSION PLANNING FOR ELECTRICITY" by Mr. Rabindranath Choudhury ,has been carried out under our supervision and that the work has not been submitted elsewhere for a degree.

(Dr. J. L. Batra)
Professor
Department of
Mechanical Engineering
Indian Institute of Technology
Kanpur - 208 016

(Dr. S. K. Gupta)
Professor
Department of
Mathematics
Indian Institute of Technology
Kanpur - 208016

August 1976

ACKNOWLEDGEMENTS

I wish to place on records my deep sense of gratitude and heartfelt thanks to both my supervisors Dr. J. L. Batra and Dr. S. K. Gupta , professors at I.I.T Kanpur for their expert and continuous guidance, valuable suggestions and useful criticisms during the course of this work. Their encouragement,motivation and inspiration throughout this work have been instrumental in getting this work completed and sustaining me through the difficult days of this work. Their pleasant and wonderful analytical abilities has inspired me.I am indebted to both of them for their immense help ,guidance and cooperation.

I am deeply grateful to Mr. T. L. Shankar, Chief of Project Appraisals Division ,Planning Commission ,Government of India, New Delhi for his valuable suggestions and also the help he offered to me to use the data available at Planning Commission, New Delhi.

I am deeply indebted to all my batchmates at IITK for their understanding and encouragement without which it would have been impossible to complete this work.

My thanks are due to Mr. J. D. Verma who has taken painstaking efforts to type the manuscript so carefully and has contributed to the style, grammar, spelling and punctuation in many places.

Numerous other people have also helped me in ways large and small. I would like to single out Mr Kripa Shankar, Mr. S.C.Padhy , Mr N.C.Sarangi, staff at the Planning Commission Library and the office staff at Project Appraisals Division ,Planning Commission, New Delhi. I am indebted and grateful to them all.

August 1976

Rabindranath Choudhury
M.Tech Student
Department of Mechanical Engineering
Indian Institute of Technology
Kanpur - 208016

TABLE OF CONTENTS

Page

Page

LIST OF TABLES

SYNOPSIS

Electricity, which is a vital input to almost all the sectors of the economy, provides an important basic infrastructure for economic development of a nation. The future developments of a country rely heavily on the rate of growth of power generation and their efficient disposal. A challenging situation is emerging out of the present inadequacies of supply and fast increasing power demands. To avoid serious repercussions on the economy due to shortage of power supply it is necessary that careful planning in the power sector is undertaken. Over all planning in the power sector involves forecasting of future demands for electricity and the development of optimal strategies for expansion of electricity generation capacities to meet the future demands.

In this thesis an attempt is made to develop quantitative models for forecasting of electricity demand and expansion of electricity generation capacities to meet the future demands. The thesis comprises of two parts. Part I deals with the development of econometric and univariate stochastic time series models for forecasting the future energy and peak power demands of electricity for India till 2000 A.D. Statistical control limits of forecasts for specified levels of significance have been estimated. Econometric models have also been developed for analysing

electricity demand in the industrial sector. The industries considered are : Iron and Steel, Non-ferrous metals, Chemicals, Vehicles, Textiles, Paper, Mining and Quarrying, Engineering and Food industries.

In part II of the thesis an analytical approach for planning for capacity expansion is presented. Planning for capacity expansion involves solving of two sub-problems in an integrated fashion, to yield minimum cost capacity expansion programme. The two sub-problems which pertain to the determination of capacities and operational strategies are solved in an iterative manner using Bender's decomposition technique to yield an overall minimum cost plan for capacity. Capacity expansion sub-problem is formulated as an integer programming model. Demand is assumed to be concentrated at load centres between which capacity and energy is to be transmitted. The size, location and type of plants are determined to meet a projected demand schedule. The operational planning sub-problems which finds minimum cost operation of the system is formulated as a non-linear programming problem with linear constraints. The feedback between the capacity expansion sub-problem and the operational planning sub-problem is repeated until the expansion sequence stops improving. The methodology has been applied to a case study and the numerical results have been presented.

CHAPTER I

INTRODUCTION

Throughout human history, the foundations of civilisation have rested heavily on supplies of various forms of energy. The nineteenth century industrial revolution was highly energy dependent, as is the process of industrialisation now underway in India. Energy has played a decisive role in the economic development of a nation. To the extent of its availability it has stimulated or hindered economic growth.

Energy is of universal use, being not only a component of productive process but also an element fundamental to welfare. Productivity is directly influenced by the amount of energy which can be incorporated into the productive process. Variations among the countries in their levels of consumption of energy is attributed to their degree of industrialisation, product mix in industry, relative importance of energy intensive processes in industrial structure, efficiency of utilisation, climatic factors and costs.

The relationships underlying economic growth and energy consumption with reference to the Indian context has been exhaustively discussed in the Energy

Survey Committee report of Government of India (1) . Planning in India is set to accomplish the objectives of removal of economic backwardness and attainment of self reliance. The suggested strategies for achieving these goals heavily rely upon the usage of energy. Davar (2) has emphasised the distinct role of energy as an effective catalyst and driving force for economic development.

The exponential growth rates of consumption of the exhaustible and non-renewable resource like energy, the four-fold increase in price of crude oil in the world market have compelled the world community to focus their attention on energy. Shortage of energy has created economic and political problems of enormous magnitude. Prompt and effective solution of these problems have become imperative to permit development to continue and to avoid threat to national stability.

1.1 Role of Electricity

Electricity is an advanced form of energy and it constitutes a major component of total energy requirements. Development of electricity has made possible to take advantages of renewable water resources. Industrial development, automation, mechanisation and urban progress have virtually become a function of electricity supply.

In the Indian context, electricity provides an important and basic infrastructure for development. It is a vital input to industry, agriculture, transport and is of particular importance to the developing rural sector which needs more and more power for its agricultural operations, small scale and agro-industries. The future development of the country depends to a great extent on the rate of growth of power generation and their efficient disposal. A challenging situation is emerging out of the present inadequacies of supply and fast increasing power demands. This calls for a careful planning in the power sector. Reliable estimates of future demands of electricity is a first step in this process.

1.2 Need For Long Term Forecasting Of Electricity

Electricity is accepted as a basic commodity and hence planning for electricity production should precede or at least closely be in harmony with planning in other sectors. The creation of production facilities for electricity involves long gestation periods. Especially this is so in developing countries with inadequate facilities for manufacturing of power plant equipment and scarcity of finances.

The investment in power sector is huge. This calls for great care and attention in making important decisions in power sector especially with limited domestic and foreign capital resources. For developing countries this necessiates projection of their future demands for electricity on a long term basis and then plan for the production facilities. A knowledge of the magnitude of demand obtained through appropriate forecasting helps decisions on planning indigeneous manufacture of equipment and complimentary facilities. For a perspective plan, demand forecasting for a time horizon of twentyfive years (spanning five five-year plans in India) would be referred to as long term projection for purposes of this study.

1.3 Forecasting Electricity Demand For Specific Power Systems

A highly aggregated demand for electricity for an entire country can be obtained by projecting the demands for individual power systems existing in a region or grid. Individual utility systems have to devise better ways to anticipate aggregate and coincident demand of all customers for planning and successful operation.

A utility system needs forecasts for a variety of purposes. The lead time and frequency of forecasts

depend on their usage. For day to day operation hourly forecasts for at least a day ahead is needed. Devising of maintenance policies require weekly forecasts with a lead time of six to twenty four months. Generation planning requires monthly forecasts upto sixty months in advance and long term corporate planning require annual forecasts for a lead time of fifteen to twenty years. In general, for an electricity supply undertaking involving both generation and distribution it can be stated that the secret of rendering better service at economical prices and reasonable profits to the organisation is to predict needs and plan ahead.

1.4 Components Of Electricity Demand Forecasts

Power system load forecasters are basically concerned with the forecasting of total electric energy requirements and peak power requirements. Energy forecasts provide a basis to estimate future revenue. Anticipated peak power demands determine company's investment in additional generation and transmission facility to ensure adequate supply.

A record of the weekly, monthly or annual peak and energy demands indicate the basic components of demand as well as the salient features of load growth. The demand pattern normally has the following components:

1. A continual growth of demand over time: This growth is attributed to an underlying component called base demand (trend component). Eventhough short variations exist, long term growth is approximated by a smooth varying trend curve.

2. Variations in demand repeating themselves with a period: These variations constitute the cyclic or seasonal component whose pattern may be preserved over the years but magnitude may grow with time by random amounts.

3. Random Variations: Superimposed on the above two components there are random components, with magnitudes much smaller than seasonal components. This is designated as noise component.

Fluctuations in the above three components depend on customer class, seasonal changes and economic activities.

1.5 Important Techno-Economic Characteristics Of An Electricity Production-Distribution System

An electric power production - distribution system has the following techno-economic characteristics.

1. Electricity is a non-storable item. Hence instantaneous power demand has to be met by instantaneous response.

2. Failure to meet demand may entail purchasing power at higher costs from a neighbouring region or system or cause serious consequences to customers.

3. Reserve capacity is expensive and unproductive. Hence a power system tries to keep a reserve as low as possible consistent with a satisfactory level of reliability of service.

4. Integration of power systems between neighbouring states or regions for economic interchange of energy and capacity is a rapidly emerging phenomenon.

5. It takes a gestation period of four to five years to plan and install generation and transmission requirements. Future corporate and systems development requires long term forecasts fifteen to twenty years in advance.

1.6 Planning For Expansion Of Electric Power Systems

With increasing automation and industrialisation it is evident that demand for an advanced form of energy like electricity would increase. While reliable estimates of future demand is an important step in planning, the responsibility of power system planners does not end here. Power system planning can be defined as a rational program for development of an electric

power system so that it can evolve in an orderly and economic manner. It includes the rationalisation of standards of service, anticipation of trends in equipment design and coordination of the various elements into a well designed whole. It is particularly concerned with plans for changes and additions to generation and transmission facilities. Summarily planning of an electric power system has to solve the problems of when and what facilities should be provided where to assure adequate service.

1.7 Need For Long-range Planning Of Power Systems

Systems planning is necessary because as the system grows existing capacity is insufficient to meet future demands. In absence of advanced planning quality of service will suffer due to long gestation periods of capacity additions. Further the enormous amount of investment in power development, inflation of costs and increasing carrying charges demand a careful planning in this sector as the costs of an incorrect plan is very high.

1.8 The Nature and Complexities Of The Task Of Planning

Modern power systems consist of a complex of installations with intricate connections. The concept of forming a grid has to come up in current practice. The process of integration effecting exchange of capacity and power entails research into these systems as a whole.

The factors of unit sizes, pattern of load growth, fuel cost charges technical and economic competitiveness of alternative plants, levels of reliability, spinning reserves, transmission cost and inflation have strong implications for planning.

The relative economics of power resources influences planning for expansion. Hydro potentials, fossil fuels and nuclear energy constitute the primary sources of power. Hydel resources are cheaper but limited in quantity and located at few places. Fossil fuel reserves are vast but thermoelectric power is economical near the source of fuel. Costs of nuclear plants are comparable but its development is restricted due to availability of nuclear fuel and radiation hazards. Technology for other forms of energy resources are far from a stage of economical exploitation. These considerations of relative economics suggest an optimal mix of hydro, thermal and nuclear plants for economic power planning in a region or grid.

For future expansion programmes decesions should be such that they fit into an overall coordinated plan that balances sizes against reserve requirements, location against transmission requirements and dates of installation against risk of loss of load.

Considering the factors stated above it is evident that determination of the optimum variants for the structural development of an electric power system is a complex and multivariate task. It is also a voluminous task on account of present day growth of interconnected systems involving large number of transmission lines and plants. The decisions pertaining to the location of plants, their type and capacity are too complex to be handled by conventional techniques. Further a planner is interested in the sensitivity of optimal plans to variety of conditions and changes in parameters. It is therefore vitally important to form a theory of optimum development of an electric power system and devise procedures for optimisation through algorithms. The determination of these optimal variants should take into account prevailing economic, technical, natural and social conditions.

The solution of this complex and intricate problem can be arrived at by the application of mathematical programming and simulation techniques with the aid of modern computers. The solutions will provide decisions regarding the location and time phasing of plants, their types and capacities.

1.9 The Characteristics Of A Power System and Their Influence On Mathematical Models

The main features of an electric power system are characterised by the following:

1. Probabilistic nature of source data : This comprises forecasts of demand, changes in technical parameters, performance of equipment and future trends of economic policy.

2. Discrete nature of capacity additions: The capacity expansion takes place in discrete steps according to unit capacities chosen for plants, the standards in force and availability of machinery and equipment in market.

3. Non-linearities of system interaction: The economic and technological link that exists between individual components of a power system are highly non-linear in nature.

The aforementioned characteristics suggest that the mathematical model of the power system will be large scale, stochastic, non-linear, discrete and dynamic. Unfortunately mathematical theory at the present stage of development cannot provide solution to this large and complex problem. Hence mathematical models to be used in practice have to be considerably simplified and systematised by using the following strategies.

(a) Linearising of principal equations expressing technical and economic relationships.

(b) Disregarding discrete nature of systems development in linear and non-linear models.

(c) Eliminating the stochastic dimensions by taking expected values of parameters and variables.

(d) Selecting basic strategies prior to solution.

(e) Determining an optimum zone of solution and making sensitivity studies to consider effects of probable variations in parameters.

The problem of planning can be considered from two points of view.

(a) Optimisation of the system as a whole.

(b) Optimisation of individual components like generation, transmission etc.

The first approach involves the development of global model which is difficult to tackle due to problem of dimensionality and limitations of present day computers. The second approach, although, permits the determination of optimal or near optimal solutions for individual components of the system, the problem that underlies this approach is the difficulty in determination of complex non-linear functional relationships linking individual components. However an attempt can be made to optimise

the development of a power system by an "order of procedure" which corresponds as closely as possible to the real process of development.

1.10 Motivation and Scope For The Present Study

In previous sections we have emphasised the role of electricity in the economic development of a nation and the consequent need and importance of forecasting future requirements and long range planning in this vital sector of the economy. It needs to be pointed out that development planning is a continuous and sequential process involving mobilisation and efficient use of resources. In this context the important aspect of development planning in the electric energy sector is the formulation of suitable policies to carryout economic activities of generation, transmission and distribution of electric energy over time and space. There emerges a need to study the problems arising in connection with rational utilisation of investment in the power sector in view of heavy capital investments involved in the development of power systems.

A survey of the literature reveals that very little work has been reported in the area of forecasting the future demands of electricity and planning for expansion of capacities of power systems in the Indian

context. Further, there exists very little evidence of the relevant applications of the sophisticated tools of econometrics, stochastic time-series analysis and theories of prediction and operational research in the field of forecasting and planning of power systems in India. These above mentioned considerations, have provided us a motivation for the present study.

This dissertation concerns itself with the development of methodologies for forecasting electricity demand and for planning the optimal expansion of capacities of power systems. Two methodologies viz. - the econometric methods and stochastic time series analysis and prediction methods have been proposed for forecasting electricity demand incorporating the probabilistic characteristics of electricity demand. Integer and non-linear mathematical programming models to be used in an iterative manner for solving the capacity expansion sub-problem and operational planning sub-problem respectively have been proposed for solving the overall capacity expansion problems of an electric power system. The models proposed have been applied to case study of an actual power system.

The present dissertation comprises of two parts. Part I of the study deals with the problem of forecasting future demands of electricity. Part II of the dissertation is devoted to the problem of planning for capacity

expansion in an integrated power system. Chapters II to VI constitutes Part I and Chapters VII to IX constitute Part II

Chapter II presents a brief review of the literature on forecasting the demands for electricity. The survey is intended to provide an overview of relevant existing work on statistical econometric and stochastic techniques of analysis and projection of demand.

Chapter III discusses the various methodologies of forecasting electricity demand and presents a relative evaluation of these methodologies. Reasons for selecting the methodologies used in this dissertation are also discussed.

Chapter IV is devoted to econometric methods of analysis and forecasting of electricity demand. Simple macro models for peak power and energy demand have been formulated and estimated. On the basis of the estimated equations point and interval forecasts of annual peak and energy demand have been obtained for India for a period of 25 years, i.e., till 2000 A.D. This chapter also presents a few econometric models for analysing industrial electricity demand. The industries considered are : Food, Chemicals, Textiles, Vehicles, Engineering, Mining and Quarrying, Paper, Iron and Steel and Non-ferrous metals.

Chapter V comprises of a brief description of the theory and methodology of univariate stochastic time series analysis and their forecasting. Time-series models have been fitted to data on past electricity demand. Forecasts based on these models have been obtained for India for a time span of twentyfive years.

Chapter VI is devoted to discussion of the results obtained by econometric and stochastic time series methodologies forecasting electricity demand. Results for analysis of industrial electricity demand are also presented. Scopes for further research into the area of electricity demand forecasting have been suggested.

In Chapter VII we present a brief survey of the literature in the field of planning for capacity expansion with special emphasis on capacity expansion planning of electric power systems. Mathematical programming and other models of power system planning have been reviewed.

In Chapter VIII we present an approach to find the minimum cost capacity expansion policies involving the determination of size, location, type of plants as well as imports and exports and long distance transmission lines between demand centres of an electric power system. A methodology for solving the capacity expansion problem and operational planning aspect of the problem in an

iterative manner is presented. The overall capacity planning problem is decomposed into two sub-problems by Bender's decomposition principle. The capacity expansion sub-problem is formulated as an integer programming model and the operational planning problem is formulated as a non-linear programming model.

The numerical results obtained by the application of the methodology presented in Chapter VIII to a case study are presented in Chapter IX. Conclusions are drawn on the basis of results obtained and scope for further work is also presented.

PART - I

FORECASTING THE DEMAND FOR ELECTRICITY

CHAPTER II

FORECASTING ELECTRICITY DEMAND - A LITERATURE REVIEW

This chapter is devoted to a brief review of the literature in the field of forecasting with particular emphasis on forecasting of electricity demand. For convenience the literature review is divided into three parts. Section 2.1 presents the relevant literature in the field of forecasting by statistical and econometric methods and their applications to forecasting the demand for electricity. Section 2.2 is devoted to the survey of the pertinent work in the area of stochastic theories of prediction and their applications to electricity demand forecasting. Section 2.3 deals with the review of other general method of projection as applied to the forecasting of electricity requirements.

2.1 Review Of Literature On Statistical and Econometric Methods Of Forecasting The Demand For Electricity

In forecasting techniques time-series and multiple regression analysis play a very important role. One popular time-series model is exponential smoothing. Exponential smoothing is based on a weighted average of two sources of evidence, one the latest (most recent)

observation and the other is a value computed one period before. As such it is an easy and quick method since very little information is needed for obtaining forecasts. Reference is made to Brown (3, 4, 5, 6, 7) and Brown and Meyer (8) who have presented an extensive description of the theory, and application of this technique to various problems of forecasting. The exponential smoothing technique along with its various applications have also been discussed by Winters(9, 10) Muth,(11) Geoffrion (12) Pegels(13) Wiener (14) , Duffin and Whiddin (15) , Duffin and Schmidt (16) , Harrison (17, 18), Harrison and Davies (19) Harrison and Scott (20) , Kirby (21) , Shiskin (22) , Holt (23,24) , Cox (25) , Buffa (26) , Buffa and Taubert (27) , Arrow et. al. (28) , Whitin (29) , Welch (30) , Magee and Boodman (31) , Moore (32) , Greene (33) , Eilon (34) , Buchan and Koenigsberg (35) and a host of others.

Theil and Wage (36) have formulated a stochastic model underlying the procedure of adaptive forecasting of an economic time-series. For a case in which the time series to be forecast has no seasonal components, they determined the weights to be used in adaptive forecasts which are optimal in the sense of minimum mean square error. The model presented by Theil and Wage is a good example of the exponential smoothing technique. The

model is expressed as

$$X_t = \bar{X}_t + S_t + \text{Residual} \ldots \qquad (2.1)$$

$$\bar{X}_t = \bar{X}_{t-1} + e_t \qquad (2.2)$$

where at time t

$\bar{X}_t$ = Trend value

e_t = Trend change

S_t = Seasonal component.

Exponential smoothing procedure predicts the trend value. At the end of the t^{th} period observation X_t is at the forecaster's disposal. <u>From equation (2.1) it is equal to $\bar{X}_t$ apart from S_t and the residual</u>.

In the absence of any information on residual and S_t, they are replaced by their expected values, which are zero and S_{t-I} respectively, where I is the length of the seasonal cycle. Now $X_t - S_{t-I}$ is the new evidence on trend level. The latest trend value $\bar{X}_{t-1}$ refers to period $t - 1$. $\bar{X}_t$ is obtained by adding e_t to $\bar{X}_{t-1}$ and e_t is replaced by e_{t-1}. This leads to the following exponential smoothing procedure for $\bar{X}_t$.

$$\bar{X}_t = \alpha(X_t - S_{t-I}) + (1 - \alpha) (\bar{X}_{t-1} + e_{t-1}) \qquad (2.3)$$

Given α, $\bar{X}_t$ can be determined in terms of most recent

observation X_t and previous computed values $\bar{X}_{t-1}$, $t-1$ and S_{t-I}.

Exponential smoothing provides procedures for detecting and adjusting to changes in forecast series rather than predicting these changes. These methods by and large do not predict in the behavioral sense. There is little attempt at explanation of causality and no information beyond the historical data of time series is used.

Nerlove and Wage (37) demonstrated that adaptive forecasting are optimal in a much wider sense. Ahtough the series generated by the Theil and Wage model is non-stationary there exists a simple transformation of the series which converts it into a stationary series. This observation permitted Nerlove and Wage to apply the Wiener - Hopf (14) theory for stationary time-series to the transformed series. Further contributions to the technique of adaptive forecasting was made by Chow (38) Dudman (39) , Wheelright and Mabridabis (40) , Jain and Batra (41) , Packer (42) , Trugg and Leach (43) , Box and Jenkins (44, 45) , Mcclain (46) , Mcclain and Thomas (47) , Morris and Glassey (48) , Buffa (49) and Griffin (50) . Most of exponential smoothing time-series models including the one given in (2.1) and (2.2) were designed to break a time series into its components

namely trend, cyclicity and noise. This allowed the forecaster to gain insight into the past history of a series through study of changes in the individual components.

Discussing monthly average load forecasting on the "Tennesse Valley Authority" system New (51) emphasised the breaking of a time-series into its components and use of seasonal indices for forecasting seasonal components. A seasonal index (obtained as a ratio of load for a given month to that of trend value for the same month) was utilised by New. The seasonal component of demand was obtained by extrapolating monthly seasonal indices and then multiplying these index of month by corresponding projected value of trend component. The computation resulted in forecast of average monthly load. The trend component was projected by extrapolating an appropriate time function fitted to load data by least squares technique.

Doobie (52) has proposed a method for the computation of coefficients even when the fitting function consists of a finite number of sine and cosine terms. Without breaking the time series into their components, suitable trigonometric functions in additions to time polynomial functions were incorporated to include seasonality. Christianse (53) used the general exponential smoothing model for short term load-forecasting of an

electric power system. Hourly loads for a power system were obtained by Gupta and Yamada (54) by adaptive short term forecasting. Weather information was utilised for determination of these forecasts.

Berry and Whiting (55) recognised the fact that trend curves must level at some stage and hence attempted to fit a logistics trend to the time series. Inspite of all possible refinements in forecasting by trend extrapolation, this methodology was critised by Cowden (56) . Cowden gave three reasons for his criticism: (1) Difficulties in finding a logical basis sufficient to justify the type of trend selected. (2) Thoughtless extrapolation can produce ridiculous results. (3) Economic and social conditions responsible for trend may not continue to be applicable outside the time-span of data and forecasting of trend component is subject to statistical errors which tends to increase as we move away from the centre of the time period to which trend curve is fitted.

The second category of forecasting techniques, multiple regression analysis, predicts a change in the forecast series through explanatory variables for a given time-series. The explanatory variables are selected on the basis of economic theory and forecaster's judgement. However unlike time-series models multiple regression models usually do not utilise information contained in

historical pattern of a time series. A regression model for forecasting electricity demand was used by Hieneman et. al. (57) who recognised the responsiveness of system loads especially to seasonal factors. Chen and Winters (58) developed a model which forecasted daily peak load of an electric power system by combining exponential smoothing and multiple linear regression models, leading to a hybrid forecasting model.

Yamada (59) extended Chen and Winter's model to the forecasting problem of a more general class by replacing the linear regression part of the model by other regression models. His general model is of the form

$$Y_t = B_t + S_t + R_t$$

$$R_t = \sum_{i=1}^{n} C_i X_{it} + \quad t$$

$$\bar{B}_t = Y_t - (\bar{S}_{t-L} + \sum_{i=1}^{n} C_i X_{it}) + (1 - \alpha) \bar{B}_{t-1}$$

$$\bar{S}_t = Y_t - (\bar{B}_t + \sum_{i=1}^{n} C_i X_{it}) + (1 - \beta) \bar{S}_{t-L}$$

Where

Y_t = Total demand at time t

B_t = Base demand at time t

R_t = Residual at time t

S_t = Seasonal demand at time t

$X_{1t}, X_{2t}, \ldots X_{nt}$ are explanatory variables

e_t is a zero mean, constant variance independent noise

α, β are smoothing constants.

L = Length of seasonal cycle.

$\bar{B}_{t-1}$ and $\bar{S}_{t-L}$ are expected values of B_{t-1} and S_{t-L} estimated at period t-1 and t-L respectively. Yamada also presented a method for identifying the parameters involved in the forecasting model.

The literature reviewed above mostly dealt with statistical and regression analysis techniques of forecasting, and their applications to electricity demand forecasting. In the following paragraphs we present a survey of the literature on econometric analysis and forecasting of electricity demand.

The general econometric techniques of forecasting has been discussed by various authors and researchers. Some of the important contributions are due to Johnston (60) , Klein (61) , Goldberger (62) , Theil (63,64,65) Dhrymes (66) , Christ (67) , Tinbergen (68) , Tintner (69) , Wannacot (70) , Leser (71) , Cramer (72) Hood and Koopmans (73) , Koopmans (74) and Wold (75) . Two good general references for forecasting economic time series are the books by Buttler and Platt (76) and Chisholm (77) . The statistical aspects of

estimating econometric models have been discussed by Maulinvad (78) . Suits (79) has demonstrated the use of an actual econometric model as a tool of forecasting for the U.S. economy. The utility of various available econometric models as instruments of forecasting were evaluated by Stekler (80) .

Enormous amount of research has been done on the development of input - output models, and their applications to forecasting. Leonlief's (81, 82, 83) . pioneering research into this area has opened up many avenues for use of this methodology. Modern contributors to the development of this technique include Morgenstern (84) , Rasmussen (85) , Stone (86) , Dorfman (87) , Dorfman, Samuelson and Solow (88) , Koopman (89) , Chenery and Clark (90) , and a host of other authors and researchers. The input - output tables for India have been discussed by Mathur and Bhardwaj (91) .

The classical theories of demand for commodities, (both capital and consumer), have been presented by Wold and Jureen (92) . All the above mentioned references on econometric methodologies (including input-output analysis) provide us techniques for projecting the demand in future for the various sections of the economy including electricity.

As far as the specific applications of econometric methods to analysis and forecasting the demand for electricity are concerned, it is observed that most of the researchers have focussed their attention on sectoral demands and especially on the domestic (residential) sector. Very few studies exist for the commercial and industrial sectors. Taylor (93) provides an evaluation and critique of the few studies in the area of econometric analysis of demand for electricity. Taylor attributes many of the problems in modelling the demand for electricity to the existence of multi-step block pricing, the fact that demand for electricity is a derived demand and the existence of distinct short-run and long-run elasticities for each class of consumer.

Classical theory of consumer demand sees the consumer as maximising a utility function defined over all goods subject to his level of income. However, while recent years have seen rapidly increasing uses of theoretically plausible demand functions (Refer Parks (94), Houthakker and Taylor (95), Phlips (96), Taylor and Weiserbs (97), Brown and Heien (98), Christensen et al. (99)) there does not exist a single econometric study of the demand for electricity for which this is the case. Demand for electricity has usually been approached in isolation or else in conjunction with the demand for its close substitutes.

Houthakker (100) has discussed the econometric implications of the existence of a price schedule. The literature has focussed very narrowly on the question of type of price - marginal or average that should be included in the demand function. Eventhough the theoretical implications of quantity discounts and block tariffs have been stated by Buchnan (101) , Gabor (102, 103) , and Oi (104) , the implications of the price schedule in the case of electricity for the equilibrium of the consumer and therefore for the demand function itself has not been systematically investigated.

Houthakker's (105) study focussed on residential electricity demand in the U.K. Using cross-section as well as time-series observations, he estimated linear and log-linear regression models, by including average money income per household, marginal price of electricity, marginal price of gas and average holding of domestic electric equipments as the explanatory variables. His results showed an income elasticity of 1.17, price elasticity of -0.89, and cross-elasticity with marginal price of gas as 0.21. Houthakker did not clarify as to whether these elasticities referred to short-run or long-run demands for electricity.

The standard and the most ambitious reference on demand for electricity in the U.S. is the monograph

of Fisher and Kaysen (29) . The authors analysed residential and industrial demand and distinguished explicitly between short-run and long-run demands. According to the authors the short-run demand was a function of utilisation rate of existing equipment while demand in the long-run was influenced by a choice of the size of the capital stock. Houthakker and Taylor (95) estimated an equation for personal consumption expenditure on electricity that was based on a state adjustment model of consumption. The short and long-run elasticities of income were 0.13 and 1.93, while that of price were -0.13 and -1.89 respectively.

Bauxter and Rees (107) have developed models explicitly for the industrial demand for electricity. Various types of fuels alongwith capital and labour were used as an input to the production functions. The demand function for electricity was derived from a Cobb - Douglas type production function. The main conclusions from their analysis was that relative price changes are not unambigyously an important determinant of growth in electricity. The chief determinants were growth in output and changes in technology. There existed a marked responsiveness of demand to relative fuel prices in some industries, while in others the price elasticity of demand was zero.

Wilson (108) analysed the residential demand for electricity. The exogenous variables incorporated were price of electricity, average price of gas, median family income, average number of rooms per household and climatic variables. The results of particular interest were substantial negative price elasticity and negative income elasticity. His results vis-a-vis price elasticity of demand are in conflict with those obtained by Fisher and Kaysen (106) . The peak load phenomenon has received great deal of attention by various authors such as Boiteux (109) Buchnan (101) , Gabor (102) , Houthakker (109) and Lewis (110) . Inspite of this the econometric literature on peak load demand appears to consist of a single study by Cargil and Meyer (111) . The regressors for estimating the demand function were the ratio of the average revenue per KWH to average price per therm of gas, real per capita income, employment of production workers in manufacturing and time. Their equation explained 90% of the variation in monthly hourly demand and indicated that price increase has a negative effect on demand, where as income is of little consequence.

In a study that in concerned more with development of methodology than obtaining results, Anderson (112) analysed producers demand for energy in the U.S. primary metals industry. Anderson's analysis, was based

on the methodology of Fisher and Kaysen, but with the following important differences:

(1) Focus was on demand for total energy rather than only electric power.

(2) Allowance was made for quantity discount of inputs.

(3) Price of input and effects of competing or related input prices were incorporated.

(4) Effects of variation in industries were considered.

The dependent variable was KWH of electricity purchased per unit of value added. Explanatory variables included were price of coal, price of electricity and oil, average wage rate of production workers in primary metals. The price elasticity of demand was negative, substantial, and highly significant.

Mount, Chapman and Tyrel (113) analysed both the short-run and long-run demand for electricity for three classes of consumers, residential, commercial and industrial. The models were estimated by pooled cross-section and time-series data of annual observations. Lagged dependent variable was used as a predictor along with population, income per capita, average price of electricity, lagged price of gas and appliances. Models for all the three sectors was estimated using ordinary least square (OLS) technique as well as the instrumental variable procedure.

The long-run elasticities demonstrated that electricity demand is generally price elastic in all sectors but income inelastic. Population exhibited approximately unit elasticity.

In another study Anderson (114) analysed the residential demand for electricity. He estimated two different models - one for predicting the stocks of energy using equipment, and the other for predicting energy consumption. The second model, involving double-log form of equation, considered income, price of various sources of energy and several demographic variables.

The demand for electric power for three major classes of consumer, residential, commercial and industrial was analysed by Lyman (115). This study contained some innovations including the use of firm data as opposed to state aggregates and use of non-linear demand functions. The non-linear demand function was in line with those developed by Box and Cox (116), Zarembka (117), and Zellner (118). Models were estimated using maximum likelihood methods. Using the variable transformation functional form Lyman suggested a linear semilogarthmic model for residential sector and double logarthmic form for commercial and industrial sector. Demands were found to be price elastic in each consumer class and residential demand was found to have a positive correlation with income.

Houthakker, Veerleger and Sheehan (119) employed a logarthmic flow adjustment model of the type used by Houthakker and Taylor (95) in analysing the demand for electricity in the residential sector. Time-series and cross-section samples of state aggregates were used. The model was estimated by using the error component technique developed by Balestra and Nerlove (120), Nerlove (121) Cicchetti and Smith (122) examined the implication of selecting alternative price measures for statistical properties of the estimated demand relationship. The criteria for selecting the best price measure was defined in form of Ramsey's tests (123,124,125). The findings for the residential sector suggested that with appropriate adjustment for simultaneity, the average price measure is preferable to a measure based on typical electricity bills.

Ashbury (126) examined the residential market with three cross-sections of fortyeight states, using average revenue as a price measure. Both OLS, and 2SLS were used. Results indicated that price elasticity estimates were stable with introduction of income, substitute prices, density and climate as explanatory variables.

Halvoresen's (128, to 131) econometric models were designed to analyse the demand for electricity in the residential sector of U.S. as well as whole of U.S.

The estimated models indicated that the long run direct elasticity of demand with regard to price was at least unitary.

Hawkins' (131) study dealt with the demand for electricity in the residential, commercial and industrial sector of New Southwales and Australian territory. Cross-section data was used. There was little evidence that commercial and industrial sector demand was responsive to price. The commercial sector demand function was interpreted as a production function with labour and services of electricity as factors. The production function showed increasing returns to scale.

The other important studies worth mentioning on analysis of electricity demand are due to Wilson (132) Tyrell(133) , Chapman, Mount and Tyrell (134).

2.2 Demand Forecasting Using Stochastic Time-Series Analysis and Theories Of Prediction

The application of probability theory to forecasting is not new. Some of the important contributions in the field are due to Hannan (135) , Nerlove and Wage (37) , Theil et al., (36), Weiner (14), Wold(136, 137) , Whittle (138) , Box and Jenkins(139,140,141,142)

Bartlett(143) , Bailey (144) , Doob (145) , Parzen (146) Anderson (147) , Ivakhenko (148) , Querouille (149) , Bartholomew (150) , Rosenblatt (151) , Kolmogrove (152) Pugachev (153) , Borde and Shannon (154) , Gabor (155) , Lubcech (156) , Crammer and Leadbetter (157) , Cox and Miller (158) , Zadeh and Ragazzini (159) , and numerous other researchers who developed the idea of stochastic prediction of a time series. Unfortunately this fast developing science of stochastic theory was never applied to forecast electricity demand. This was natural because except for a few research papers by Box (139), Nerlove (37), Theil (36) and Whittle (138) the stochastic prediction theory was almost universally applied to stationary time series. Modern stochastic approach to problem of filtering and prediction was launched by Wiener (14) in his now classical work. He developed techniques for synthesis of optimal linear systems for filtering and prediction of stationary time series. Wiener showed that the linear filter was the absolute optimum filter for Gaussian noise under minimum mean square error criterion. Wold (136) carried out the idea of linear filter forward and stated that any stationary time series could be considered as an output of a linear filter with white noise as input. Given a time series a linear filter given by its impulse respose function could be derived, which has its output the

desired forecasts. Whittle(138) used the concept of 'Z' transform in order to estimate the impulse response function of the prediction filter based on Wold's theory. Use of Z transform simplified mathematical computation and made prediction scheme very elegant. Carrying forward the idea of stationary time series he formulated a prediction scheme for those types of non-stationary series which could be reduced to a stationary time series by finite linear transformation. Box and Jenkins (44) introduced a practical prediction scheme for handling non-stationary time-series and showed it to be optimal for a particular class of stochastic process. While most literature dealt with the problem in the time domain, an occasional try was made to define a Power spectrum of a non-stationary time series. Priestley (160) developed an approach for spectral analysis of non-stationary time-series, which was based on the concept of evolutionary spectra i.e., a spectral function which was time dependent and has a physical interpretation as local energy distribution over frequency. Inspite of Priestley's elegant mathematical analysis, the method was to divide the series into segments, compute spectra, consider each segment as stationary and then extrapolate the evolving value of spectra. In a paper Rao and Shapiro (161) proposed an alternative scheme for using the evolutionary spectra in adaptive smoothing. The smoothing

constants were determined as a function of maximum change in various frequency components of successive spectra. The location and type of change indicated the disturbance in the underlying stochastic process generating the series. Literature cited above has been mostly theoretical but Theil and Wage (36) formulated a stochastic model underlying the procedure of adaptive forecasting of an economic time-series. Nerlove and Wage suggested that given the same underlying model adaptive forecasting was optimal in a much wider sense. Couts et. al.(162) carried forward the general idea of Nerlove and Wage for predicting those type of non-stationary series which can be reduced to a stationary series by finite linear transformation. Farmer (163) has presented the application of the theory non-stationary time-series prediction to electricity demand estimation. The method utilised an adaptive time-series approach representing the load cycle in terms of characteristic function.

New (164) appears to be one of the first investigators to add probability dimension to the forecasting of electricity demand. He decomposed the time series into its components viz. cyclic, trend and noise. Forecasts and the associated probability distribution for each component were obtained seperately. The Monte-Carlo gaming technique proposed by Hammersley and Handscombe (165) was then used to combine the probability distribution of

components to get the final forecast. He claimed that the use of the Monte-Carlo gaming technique offers certain advantages. The advantages accrue due to the fact that the technique does not require complex convolution scheme to combine composite distributions. Latham et. al. (166) have suggested a procedure for the integration of various probabilistic component forecasts into the total forecast. The integration was obtained through a subjective distribution method. This procedure is not very sound in the sense that different forecasters would obtain different forecast distributions from the same data.

Stanton and Gupta et. al(167,168,169,170)have applied stochastic time series models to long range and short range forecasting of demand for electricity.

In a dissertation Gupta (171) has applied the stochastic theory of prediction to forecast the weekly, monthly and annual peak demands in an utility system.

Some state estimation type of modelling for load forecasting were used by Toyoda and associates(172,173,174, 175,176,177).The sequential filter technique suggested by Schweeppe and Wildes (178)and Larson et. al.(179) were used by Toyoda et. al. for this short term forecasting. The sequential filtering techniques for forecasting were proposed by Kalman and Bucy (180) , Sorensen (181) , and Aoki (182) , Sage and Husa, and Mehra (183,184,185).

2.3 Other Methods Of Forecasting The Demand for Electricity

The previous sections presented a review of the literature on forecasting by statistical, econometric, and stochastic time series methods. In this section we shall present a few methods, other than the methods reviewed in the previous sections, which have been utilized for forecasting of electricity demand.

Hooke (186) proposed a method for forecasting the demand for electricity by class of service for a time interval of three to five years. A secular trend was drawn from past data to forecast conditions three to five years hence. An estimate was made of expected deviations from trend in the near term future. By summing up the estimate of sales in each category and then adding losses, a forecast was made. Expected peak demands were determined by a process of correlating peaks with consumptions and system output.

Goddard (187) determined future electricity energy requirements by estimating the sales in each category of demand namely residential, commercial and industrial. For each class the demand was estimated by relating the consumption to some economic and demographic variables.

Schiller (188) studied the demand pattern of different class of consumers by regression analysis techniques. In Electricite' de' France demand forecasting has been the subject of several studies(189,190,191,192) These studies have emphasised that growth in demand for electricity is closely tied up with technical progress, and economic expansion. The probability distribution of future consumption has been expressed as a function of the rate of general economic development as forecasted in the process of national planning. Models representing successive demands increments by a Markov process have given satisfactory estimates of global demand.

Arnoff and Chambers (193) have advocated the determination of peak power demand from estimated KWH sales, since the error in estimating annual sales is normally less than error in estimating peak demand.

The effect of marginal cost pricing for large customers, on peak demand has been studied by EDF (194, 195) . The studies indicated that there was successive improvement in annual load factor due to adoption of marginal cost pricing principle.

The study by NCAER (196) on determination of the future requirements of electricity for India has applied the'method of end uses' techniques. The future

demand has been obtained by projecting the outputs in various electricity consuming sectors and then determining the total requirements of electricity from the knowledge of specific electricity requirements per unit of output. Similar techniques have been applied by the energy survey committee of the Government of India to forecast future electricity requirements.

A mathematical formula requiring the use of population forecast only has been developed by Scheer (197) of U.S. The formula has been developed from the thesis that for every hundred fold increase in per capita generation the rate of growth in generation will be reduced by half. This formula has been used for forecasting generation requirements in utilities.

Mukhopadhyah and Tripathi (198) have advocated the use of flow graph analysis techniques to load forecasting. Taking economic prosperity as a starting point a flow graph may be drawn relating growth of demand for power due to the urge for economic prosperity.

Methods for handling weather sensitive demand has been discussed by several authors including Hieneman (57) , Thompson (199) , Davey et. al. (200) Ashbury (201) , Clair and Einwefcher (202) , Corpening et. al. (203) . The peak demands and energy are separated to weather sensitive and non-weather sensitive

Page number 42 has been omitted by mistake in the sequence of page numbers.
Please read page number 43 in continuation of the matter in page number 41.

component. Weather models are determined by correlating demand with appropriate weather variables. Weather models are determined from data sampled more frequently.

The literature survey presented in this chapter is not claimed to be exhaustive. Forecasting techniques are in an area of gradual development and more and more research is being done on this area. However the more advanced techniques of forecasting to a large extent make use of a combination of simple approaches with modern developments of prediction theory.

A survey of the presented literature reveals that although considerable amount of research has been done in the area of forecasting the demand for electricity on the international scene, very little work seems to have been done in this area in the Indian context. This has motivated us to investigate and formalise strategies for long range probabilistic forecasting of annual peak demand and energy for India. In Chapter III, the methodologies selected for projecting demand for electricity in the present study have been presented.

CHAPTER III

METHODOLOGIES OF ELECTRICITY DEMAND FORECASTING THEIR EVALUATION AND SELECTION

3.1 METHODOLOGIES OF FORECASTING ELECTRICITY DEMAND

The main types of techniques used for projecting electricity demand can be divided into the following groups:

1. Direct extrapolation of rates of change in requirements on the basis of observed characteristics of a past trend.

2. Extrapolations from observed relationships based on some deduced functional relationships and causality between variables.

3. Application of statistical and stochastic time - series analysis and theories of prediction.

4. Methods of comprehensive plans - Extrapolation of specific energy requirements per unit of product to planned production.

5. Input - output techniques.

6. Comparative international analysis - Evaluation of future development from comparative study of its homologues in different countries.

In the following paragraphs we shall discuss in brief the main features of the aforementioned methods and present an evaluation of these methods.

3.1.1 Direct Extrapolation On The Basis Of Observed Characteristics Of A Past Trend

This procedure is widely used for short term projections (up to one or two years ahead) of electricity demand. The basic assumption in all these methods of forecasting by trend extrapolation is:influences those were at work in the past will continue to exert a similarly evolving total effect in future. Two distinct variations of the method may be envisaged. They are:

1. Extrapolations may be carried out from a rate of change deduced from past statistical data.

2. Extrapolation by application of a mathematical formula which fits development in the past and is assumed to characterise the likely changes in future.

To the same general class of mathematical extrapolation techniques belongs, an attempt to fit an 'S' curve derived from expressions similar to a logistic trend, which accelerates to a point of inflexion, beyond which growth declines slowly and then becomes asymptotic. In

general straightline, parabolic, exponential and modified exponential curves are applied. Out of these the one which appropriately reflects the growth is selected.

Extrapolations from rates of change in past time series can be obtained from matrix tables which sum up all rates over a specified period. Such methods normally assume the retention of a fair degree of stability of demand. Extrapolation can also be carried out using graphical methods. Graphical methods include assembling a series of long term trends as established from consecutive years in the past and plotting them all together from the same point as origin. The resulting cluster of graphs are summarised by plotting the median curve and upper and lower quartiles. This method is normally used as a starting point for other extrapolation techniques.

3.1.2 Extrapolations From Functional Relationships

This mode of projection inspite of its many variants offer a distinct and widely used extrapolation method. Electricity requirements have significantly high correlations with certain economic variables. The deduced functional relationships between demand for electricity and these variables provide the basis for extrapolation of demand.

Study of the many types of associative formulae shows that economic indicators of the following types are most frequently incorporated in the relationships.

1. Demographic data of -

   (a) Total population.

   (b) Total labour force.

   (c) Employment in industry.

2. Economic data such as

   (a) Gross national product or national income.

   (b) Industrial production.

   (c) Fixed capital formation.

   (d) Per-capita income.

   (e) Steel consumption.

   (f) Agricultural output.

   (g) Transportation indices.

   (h) Indices of commercial activity.

The multiple regression technique which expresses linear and non-linear functional relationships is generally employed to link the demand for electricity with the above variables. The coefficients of such relationships represent the effect of unit change in one variable on the variable to be forecasted.

Even while restricting the approach to linear regression equations based on method of least squares,

many variations are possible. The values of variables may be specific (or absolute) or they may be represented by indices. Further, they can be expressed in linear coordinates or in logarthmic form. It is evident that the periods for which functional relationships are estimated should not be too long, if structural changes in parameters are likely to occur.

3.1.3 Forecasting By Stochastic Time-Series Analysis And Theories Of Prediction

Any set of observations arranged chronologically constitute a time-series. The basic idea of stochastic theory of time-series analysis is to regard the time series as an observation made on an ensemble of random variables. Assuming that the peak power and energy demand are stochastic and stationary in nature the principles of time-series analysis can be applied to forecast electricity demand. Non-stationarity of data can be handled by converting them to a stationary series by means of appropriate transformations. The auto-correlation and partial auto-correlation function are used as tools.to identify the nature of persistence. Cyclicity in the data, if any, are revealed and identified by applications of spectral analysis techniques. Having identified the trend, auto-regressive, moving average, and cyclic components in the time-series of past data and the order of

the stochastic process, the parameters of the model are estimated by refined statistical iterative techniques. After validation of the model to be fitted by means of diagnostic checking, the stochastic theories of prediction can be applied to forecast future values of demand and obtain their range for a given level of significance.

3.1.4 Comprehensive Economic Programmes And Aggregative Forecasts

This method of projection seeks to cover the entire field of electricity supply and demand, where as the three methods discussed above are mainly adapted to determination of macro-level forecasts. Energy requirements for a given economic system should be determined comprehensively so that the requirements of different consuming sectors are mutually consistent and in line with expected trends in conversion efficiency and substitution. Such estimates allow for effect of technical progress and social developments, which initiate structural changes in the growth of industries.

Centrally directed plans and projections which adopt a comprehensive approach in respect of electricity requirements include the one-year, five-year and long term or perspective plans. Requirements for electricity in the principal industries and consuming sectors are

worked out by applying to planned levels of output, the specific electricity input per unit of output. Seperate sectoral estimates are aggregated and they are harmonised with local and regional estimates. An organised study of the probable interplay of factors making for substitution replacement is essential for the comprehensive method of forecasting.

The method of approach in preparation of comprehensive forecasts relies on the following three distinct type of inputs.

1. Assembly of local and regional extrapolations by supply undertakings based on detailed special knowledge of requirements.
2. Careful studies by individual consuming sectors (market studies in residential sector, production forecasts in industrial sector, etc.).
3. Macro studies linking electricity requirements with indices of economic growth.

The comprehensive forecasts for electricity demand is based on the reconcilation of the information obtained from the above mentioned inputs.

3.1.5 Forecasting By Input-Output Techniques

The input-output model is represented by a system of simultaneous equations which link output of all sectors and industries to the inputs used in those sectors. It is utilised to analyse and predict all the productive inter-industry transactions (by industry categories) and others that go into determination of gross national product. The input-output methodology provides a means by which economic forecasters can convert their estimates of aggregate final demand to estimates of total required output in every sector including electricity and amount of resource inputs that are consistent with GNP projections.

3.1.6 Projections By Comparative Analysis Of Homologue Trends In Different Countries

This technique involves the comparison of trends of long term and short term growth rates and their changes in different countries of similar economic structure. Expressions giving first an accelerating and then a declining growth on the model of the logistic curve are observed to fit trends in electricity consumption. Hypothesis governing future changes in growth rates are formulated from observed relationship between rates of growth and electricity consumption per head. A simple illustration of the method is to determine the correlation between national

CENT[illegible] [illegible]RY
Acc. No. A. 112190

income and electricity consumption per capita for a wide range of countries. This information can be utilised for the forecasting of electricity demand.

3.2 AN EVALUATION OF THE METHODOLOGIES OF ELECTRICITY DEMAND FORECASTING

In the previous section we have presented six different techniques of forecasting electricity demand. These techniques do not necessarily exclude one another completely. They all reduce to some logically consistent mode of extrapolation offered by observed regularities of past behaviour. The inherent assumptions are the continuation of the mode of evolution of factors generating the demand for electricity. In each technique provisions have to be made for uncertainity and unanalysed interplay of various technical, social and economic factors by use of informed judgement. It has been observed that despite some common features these methods of projection necessarily differ widely in their application and use. Each method has its distinct advantages, disadvantages and problems of application.

The two most important assumptions in the trend extrapolation approach are that the growth process continues to remain same throughout the historical period and that the factors responsible for growth continue to evolve

in the same manner. These assumptions distinctly appear to be implausible. There can be no assurance that forces which influenced past growth will continue unabated and no new factors will be introduced. Ragnar Frisch (204) an eminent econometrician calls the trend extrapolation method, the most primitive technique of forecasting. It needs to be pointed out that the trend extrapolation method does not generally take into consideration major structural and investment changes in the economy. Cases may exist when during past years there might have been unusually high rates of growth due to a low datum of electricity consumption. Extrapolation from such high rates, which usually are non-linear, will result in unusually high estimates of demand, which may not at all be realised. Hence results of trend method need to be modified and revised by exercising a great deal of judgement in anticipating future conditions. The application is limited to short range forecasting. Despite these limitations trend extrapolation technique certainly is simple and easy-to-apply. It has the advantage and utility in providing a convenient first approximation.

The difficulties which underlie the econometric methods are numerous. The structuring and formulation of a complete and detailed simultaneous equations econometric model representing the realistic linear and nonlinear and complex technical, economic and behavioral relationships

is a difficult task. There always exists a possibility that the assumed form of functional relationships do not represent the forecasting situation realistically. In addition there are problems of identification, specification, choice of appropriate independent causal variables and errors in data and their measurements. The inherent and fundamental assumptions in the methodology, of non existence of multicollinearity in independent variables and autocorrelation in errors, which are difficult to realise, create problems of estimation. Even though long series of data are advantageous and desirable for purposes of estimation, the distinct possibilities of structural changes over time may not be ruled out. Parameters and coefficients may turn out to be unstable and non-stationary, requiring frequent revisions. The forecasts obtained by the application of this method can only be relied upon if the structural coefficients are stationary, proper regressors are choosen and reliable estimates of future values of the stochastic regressors exist. Since the econometric models are essentially static in nature, forecasts need fairly frequent amendments. Notwithstanding above problems, econometric methods are emerging as excellent tools of forecasting the demand for electricity, since they explicitly take into account the significant causal relations that exist between electricity demand and economic variables.

The facility afforded by stochastic time-series analysis approach for choice of a forecast function appropriate to the particular problem under consideration is the most distinctive features of this method. Nevertheless it is just this characteristic which has led in some instances to criticism of the approach and difficulties in its practical application. The method provides a class of models and strategies by which particular models can be choosen from the various classes which reflect the properties of individual time series. The eventual forecast function is dictated to a large extent by the data used. The models capture the totality of information coming out of the data. Further, in contrast to typical exponential smoothing techniques, this approach allows a good deal of freedom and judgement in the choice of an appropriate model. The method has been criticised on grounds of requirements of large amount of time for analysis and need of better expertise. The freedom of choice offered by the approach may lead to the selection of a poor model producing forecasts of low quality. The method does not guarantee numerically identical conclusions from the same set of data by different analysts. A moderately long series of data are required for its satisfactory application. Sophisticated and complex computational procedures are needed for model identification, estimation and

contemplated since there exists several potential areas where these models are likely to find applications.

The method of comprehensive forecasts is an aggregating, compensating and cross-checking approach which replaces the reliance on the relative assurance of an economic plan. It uses overall economic forecasting combined with aggregation of a large number of sectoral and regional estimates of electricity needs. However, for all countries the long term development and structural economic changes pose problems which cannot be forecasted once and for all. Perspective plans are controlled by macro-level projections prepared with variations allowed for alternate developments. The formulation of energy needs can only be arrived at in stages since they are mainly derived from output plans. Underlying this methodology are the problems of obtaining data and predicting the specific needs per unit of output in future.

Projection by comparative analysis of homologue trends is by far the least well understood and documented of projection techniques. The major problem in using this methodology arises due to the uncertainities involved in forecasting the nature of long term economic development. The use of the comparative method to study demand relationships in different countries and to make projections based on these relationships, aids in

obtaining improved forecasts by reducing the apparent dispersion between tendencies in different countries and by allowing projections to be made more realistically. This however is only true if the data choosen are linked in a related development process. Such a development process can be discerned in the world phenomenon of characteristic growth rates for electric energy, which might be explained in terms of changing rates of capital investment and evolving levels of labour productivity and personal consumption. The value of this methodology is implied in the emphasis given to international comparability in various surveys of electricity demand. Characteristics varying with structure of national resources and the resulting pattern of industrial structure in different countries are less likely to yield useful results except where their structural features can be matched. The difficulties that underlie the application of this methodology is in procuring, organising and co-ordinating the international data.

A comparative/relative evaluation of the methodologies presented above lead us to the conclusion that none of the methodologies are entirely satisfactory and fool proof. The usefulness of any particular method rests on the nature of problem in question, availability of data and considerations of time and resources.

3.3 SELECTION OF METHODOLOGIES FOR THE PRESENT STUDY

The selection of one or more methodologies for forecasting is influenced by various factors. Amongst several considerations the availability of suitable data for the past, and their time span are major factors, influencing the choice of a methodology. An ideal projection technique should take into account the influences of many interrelated factors of rates of growth, of changes in structure of activities and technical progress; since projections allowing for these factors can be checked for consistency. Further, due to the uncertainities involved in forecasting, forecasts obtained by the application of any methodology should be specified for a given probability level by defining upper and lower limits of forecast. An additional factor that should be given due attention while selecting a methodology for a given forecasting situation is the adaptability of the methodology to the nature of forecasts.

A relative evaluation of the methodologies indicate that none of the methods of projection are entirely satisfactory for a given application. However, after careful consideration and evaluation of the relative merits and demerits of the six methodologies presented we have selected the following two methodologies for

the purpose of the present study. They are

(1) Econometric methodology

(2) Stochastic time-series analysis and forecasting methodology.

In the following paragraphs we provide reasons for the selection of these methodologies.

3.3.1 Reasons For Selecting The Econometric Method For Electricity Demand Forecasting

It is evident that the demand for electricity has a significantly high correlation with the levels of economic and industrial activity as well as with certain demographic variables. Macroeconomic theory provides us the overall frenework for forecasting an economic variable (like electricity consumption), by providing a sound hypothesis which explains economic behaviour, and based on which causal variables can be chosen.. It is the econometric method which explicitly considers the functional relationship between causal economic variables and the demand for electricity,and also provides a procedure for quantitative estimation of these relationships. Further it needs to be pointed out that the implications of any policy, which is normally required for planning and devising policies, can only be determined by formulation of a model which recognises the effects and interactions of

economic variables on demand for electricity. The application of the method of extrapolating the demand for electricity, from quantitative econometric relationships are rare in the Indian context and projections made explicitly as probabilistic statements referring to a particular point in time are mostly unavailable. It is these aforementioned reasons, along with the availability of reasonably good quality data, which has motivated us to use this technique for electricity demand forecasting.

3.3.2 Reasons For Selecting The Stochastic Time-Series Analysis And Forecasting Technique

The following considerations have provided us a motivation to use this methodology for electricity demand forecasting.

The theory of stochastic time-series analysis proposes a general class of models out of which particular models can be choosen according to the properties of the individual time-series data of peak and energy demands. The eventual forecast function is to a large extent dictated by the data, a property referred to as "letting the data speak for itself". The method captures the totality of information that can be derived out of a time series. As pointed out earlier, the present method offers a good

deal of freedom and choice to exercise judgement in selection of appropriate models. With the availability of more data models can be modified and revised through a more thorough analysis.

Stochastic methods involve the development of probabilistic models which have as their output the evolving electricity demand. This methodology offers us models possessing maximum simplicity and minimum number of parameters concordant with representational adequacy. The obtaining of such models may be important as they may provide us information about nature of the system generating the time-series. Further once an appropriate model is built optimal forecasts can be prepared rapidly from past and current data.

In the present dissertation the above mentioned two methodologies have been applied to the problem of electricity demand forecasting for India. The details regarding the econometric and stochastic time-series analysis and forecasting methodologies have been presented in Chapter IV and Chapter V respectively.

CHAPTER IV

FORECASTING OF ELECTRICITY DEMAND BY ECONOMETRIC METHODOLOGY

This chapter is devoted to the development of models for forecasting the future requirements of electricity by the application of econometric methodology. We propose macrolevel econometric models to analyse and estimate the demand function for electricity. The models have been applied to forecast future requirements of electricity in India for the post sample period (from 1976 to 2000 A.D.). In section 4.1 we present the macro econometric model for forecasting future peak demands and energy requirements. Section 4.2 deals with the formulation and estimation of models for analysis of industrial demand for electricity. Results obtained by the application of the methodologies and their analysis are presented in Chapter VI.

4.1 EXOGENOUS VARIABLES AFFECTING THE DEMAND FOR ELECTRICITY

Electricity being a factor of production has a derived demand. By principles of economic theory we postulate that the demand for electricity in an economy depends positively on the measures of economic activity, which use electricity. Economic and demographic variables

which are expected to have a significantly high correlation with demand for electricity are gross national product (national income), real per capita income, price of electricity and price of their substitutes, industrial output, agricultural production, transportation activities etc. Electricity is utilised in all sectors of the economy and coal, fuel oil, gas and other forms of energy resources are used as its substitutes.

The choice of macro causal variables (which are to be used as independent regressors) effecting electricity demand can be confined to gross national product, industrial output, population, per capita income, price of electricity (relative price) because variables like agricultural production, transportation activities are indirectly included in gross national product values.

4.1.1 Econometric Methodology - Its Tools And Problems

Before presenting the actual models, we discuss in brief the econometric methodology - its tools and problems.

Three distinguished economists, Samuelson, Koopmans and Stone (205) have ably described econometrics as a quantitative analysis of actual economic behaviour based on concurrent development of theory and observation

related by appropriate methods of inference. Econometric studies involve systematic collection of information about actual phenomenon and construction of empirical correlates to theoretical concepts. This activity draws mainly on mathematical statistics and economic theory. The objective of an econometric model is to obtain quantitative statements that explains the behaviour of variables already observed and forecast their future values. Econometric models are essentially stochastic whose distinguishing feature is the presence of a disturbance or error term. The models are generally in form of single or a system of linear and non-linear equalities and inequalities with numerically specified coefficients. An ideal econometric model should have properties of relevance, simplicity, theoretical plausibility, explanatory ability, consistency and accuracy of coefficients and forecasting ability.

The econometric model building and its validation is generally accomplished by

1) stating the problem clearly,
2) choosing an appropriate model or maintained hypothesis to be used for the problem,
3) observing relevant data, and
4) using statistical inference techniques based on the maintained hypothesis and drawing inferences about the problem from data.

The difficulties underlying the above mode of attack are to obtain accurate knowledge about the appropriate model. The main source of maintained hypothesis is economic theory which at best specifies the conceptual variables to appear in an equation, the algebraic sign and value of certain partial derivatives and the functional form, but this is not possible in all situations. In such an eventuality it is recommended to choose a functional form of equation which is reasonable on grounds that are in part theoretical and empirical. One choice is to try out several different theoretically plausible functional forms in a sort of experimental fashion and select the one amongst them which fits the data well. However this method is beset, with certain problems. These problems stem from statistical inference and description of temporary or accidental features of data rather than enduring systematic features.

4.1.2 Development Of Econometric Forecasting Model For Electricity Demand

We specify the following implicit functional relationships between electricity demand and the chosen independent variables.

$$E_t = f_1 \, (GNP_t, \; I_t, \; P_t, \; Pe_t, \; Ps_t, \; t) \qquad (4.1)$$

$$PD_t = f_2 (GNP_t, I_t, P_t, Pe_t, Ps_t, t) \qquad (4.2)$$

Where,

E_t = Annual energy consumption (demand) in year t in million KWH.

PD_t = Peak demand in year t in MW.

GNP_t = Index number of gross national product in year t.

I_t = Index number of industrial output in year t.

P_t = Population in year t (Millions).

Pe_t = Price index of electricity in year t.

Ps_t = Relative price index of substitutes of electricity in year t.

In the above relationships it is assumed that all money oriented indices are deflated by a suitable price index to neutralise effects of inflation in the variables. This would result in indices at constant prices. Demand in this study is defined to be identical to consumption or satisfied (effective) demand.

The demand functions specified by equation (4.1) and (4.2) are implicit. The various econometric models studied are given below. A relationship linear in parameters and variables has been assumed. Four variations of the linear model are proposed. The first

model is of the simple linear form. Model 2 incorporates first differences of variables. The third model is of the double log-linear form. Model 4 incorporates first differences of log transformed variables. Models proposed for peak power and energy demand are identical. The models are as follows:

Model 1

$$\begin{aligned} E_t &= \beta_0 + \beta_1 \, GNP_t + \beta_2 \, I_t + \beta_3 \, P_t + \beta_4 \, Pe_t \\ &\quad + \beta_5 \, Ps_t + \beta_6 \, t + \epsilon_t \\ PD_t &= \beta_0' + \beta_1' \, GNP_t + \beta_2' \, I_t + \beta_3' \, P_t + \beta_4' \, Pe_t \\ &\quad + \beta_5' \, Ps_t + \beta_6' \, t + \epsilon_t' \end{aligned} \tag{4.3}$$

Model 2

$$\begin{aligned} \Delta E_t &= \beta_0'' + \beta_1'' \, \Delta GNP_t + \beta_2'' \, \Delta I_t + \beta_3'' \, \Delta P_t \\ &\quad + \beta_4'' \, \Delta Pe_t + \beta_5'' \, \Delta Ps_t + \beta_6'' \, t + \epsilon_t'' \\ \Delta PD_t &= \tilde{\beta}_0 + \tilde{\beta}_1 \, \Delta GNP_t + \tilde{\beta}_2 \, \Delta I_t + \tilde{\beta}_3 \, \Delta P_t \\ &\quad + \tilde{\beta}_4 \, \Delta Pe_t + \tilde{\beta}_5 \, \Delta Ps_t + \tilde{\beta}_6 \, t + \tilde{\epsilon}_t \end{aligned} \tag{4.4}$$

Model 3

$$\begin{aligned} \text{Log } E_t &= a_0 + a_1 \log GNP_t + a_2 \log I_t + a_3 \log P_t \\ &\quad + a_4 \log Pe_t + a_5 \log Ps_t + a_6 \, t + \xi_t \\ \text{Log } PD_t &= a_0' + a_1' \log GNP_t + a_2' \log I_t + a_3' \log p_t \\ &\quad + a_4' \log Pe_t + a_5' \log Ps_t + a_6' \, t + \xi_t' \end{aligned} \tag{4.5}$$

Model 4

$$\Delta \log E_t = b_o + b_1 \ \Delta \log GNP_t + b_2 \ \Delta \log I_t + b_3 \ \Delta \log P_t + b_4 \ \Delta \log Pe_t + b_5 \ \Delta \log Ps_t + b_6 \ t + u_t$$

$$\Delta \log PD_t = b_o' + b_1' \ \Delta \log GNP_t + b_2' \ \Delta \log I_t + b_3' \ \Delta \log P_t + b_4' \ \Delta \log Pe_t + b_5' \ \Delta \log Ps_t + b_6' \ t + u_t' \quad (4.6)$$

where,

Δ represents first differences of variables,

β, a, b, etc. are the coefficients of models to be estimated, and

ϵ, ξ, u, etc. are the error terms.

The models proposed above include all the independent variables. Various other models can be formed with alternative combinations of variables. The time variable is included as a surrogate for all time-trended variables like rate of electrification, and technological progress, etc. No published data on electricity price and prices for substitutes of electricity are available. However, data on relative price of electricity with respect to fuel price index are available. Therefore, the variables Pe_t, and Ps_t are replaced by the relative price index, since it is construed that this ratio reflects price of electricity, and other fuels. Further as most of the

variables are significantly time-trended due attention has to be given to the problem of multicollinearity in independent variables. The problems of statistical estimation in presence of multicollinearity and autocorrelated error terms have been discussed by Johnston (60), Theil (65), Tintner (69) and others.

4.1.3 Data Used For The Problem And The Estimation Procedure

The time series data for the period 1928 to 1975 on electricity consumption, peak demand, GNP, population, industrial output, relative price of electricity have been collected from various sources. The sources of the data are listed in Appendix. Except population all other data for the independent variables are expressed in terms of indices with base of 1961 - 1962 equal to hundred. The data on electricity consumption are expressed in million KWH units and peak demand in Megawatts.

The ordinary least squares technique has been used to estimate the coefficients of the equations. A computer program has been written in Fortran IV based on the linear least squares algorithm.

age number 71 has been omitted by mistake in the sequence of page numbers.
Please read page number 72 in continuation of the matter in page number 70.

4.1.4 Forecasting The Future Values Of Electricity Energy And Peak Demand Based On Estimated Functional Relations

To obtain reliable forecasts of future requirements of electricity energy and peak demand on the basis of an estimated relationship we need to have reliable estimates of future values of independent variables. Reliable estimation of future values of GNP, population, industrial output is a difficult task, as these variables depend on a variety of economic, technical and social factors which are highly stochastic in nature. Methods for prediction of economic variables have been discussed by Klein (61) , Theil (63,64) , Buttler et. al. (76) and various other authors. The application of these techniques for prediction of future economic and demographic variables are beyond the scope of this work.

We have resolved the problem of estimating the future values of relevant exogenous variables in a relatively simpler manner. It is assumed that the past rates of growth in these variables will be maintained in future. The future values of the independent values are extrapolated from these past growth rates. Another choice is to use the planned and anticipated growth rates for these variables for extrapolating their future values. Substitution of these future values in the estimated relationship provides us the point forecasts. The upper and

lower ranges for these forecasts have been obtained for a given probability level.

The values of forecasts obtained for India for the period 1976 - 2000 are presented in Chapter VI.

4.2 ANALYSIS OF INDUSTRIAL DEMAND FOR ELECTRICITY

In this section we present econometric models for analysing the demand for electricity in the industrial sector. The approach adopted involves construction of models incorporating hypothesis based on accepted economic principles. The quantitative estimates of various structural coefficients are obtained using the historical data. The objective of this analysis is to obtain a clear understanding of the basic factors influencing past developments of industrial electricity demand. The proposed models may be used for forecasting electricity demand for the industrial sector.

The analysis has been carried out for the following groups of industries viz. Iron and Steel, Textiles, Paper, Vehicles, Engineering, Mining and Quarrying, Nonferrous Metals, Chemicals and Food in Indian context. The various models proposed, are based on postulates, which describe relationships between relevant exogenous variables and demand relying on macroeconomic principles.

4.2.1 Formulation Of Hypothesis About Determinants Of Demand For Electricity In The Industrial Sectors

The approach adopted in this study is to treat electricity as an input entering into the production function. Using this philosophy we develop specific hypothesis about the form of relationships involved and particular variables which appear to be relevant. We will describe three basic models each derived from a differing primary hypothesis.

In the industrial sector the main features of electricity consumption in the past has been that consumption of electricity grew faster than the output in every industry. The following three explanations can be put forth for this phenomenon.

1. Relative price movements may induce substitution of electricity for other fuel inputs and possibly labour.
2. Technological changes may be such that they lead to innovation based on use of electricity. Further technological changes may also induce substitution of electricity against other fuels and labour. This substitution is attributed to the technical advancements in the usage of electricity rather than a favourable price movement for electricity.

3. The normal explanation given by economic theory for non-proportional variation in input and output is the existence of varying returns to scale. Electric power does not fit into this framework since its relationship with output is indirect through the medium of plant and machinery. The demand for electricity is a derived demand and hence it is effected by the demand for output of all electricity using commodities. In the long run therefore the variation of electricity demand with variation in output may depend on the following two factors.

i) Factors which determine how capital stock varies as output varies.

ii) Factors which determine the electricity using characteristics of increments in capital stock.

Thus when we observe that electricity demand grows faster than industrial output, this would suggest the following:

(a) Greater proportional increase in capital stock than in output might have led to faster growth of electricity demand.

(b) The electric intensity of the additional capital stock needed for increased output might be above the average for the existing stock.

Thus when relative full price and technological change do not appear to have influenced the growth of electricity demand, faster growth of electricity demand as compared to output may be due to more than proportionate increase in capital stock as compared to output. The absence of adequate data inhibits the explicit inclusion of electricity intensity and capital stock relationships in the development of forecasting models.

The three models proposed in the following section are based on hypotheses, which use the first two explanations given in this section for the higher rate of consumption of electricity as compared to the industrial output.

4.2.2 Models For Industrial Electricity Demand

Model 1

The development of this model is based on the straightforward application of the theory of demands for inputs. We assume that firms have a production function of the Cobb - Douglas (206) type, defined on inputs such as labour, capital, raw materials and various forms of energy. Let the output of an industry be Q. Then

$$Q = \alpha_0 \; x_1^{\alpha_1} \, x_2^{\alpha_2} \; \cdots \; x_k^{\alpha_k} \qquad (4.7)$$

where

x_j = relevant inputs, $j = 1, 2, \ldots k$

α_j = parameters for x_j, $j = 1, 2, \ldots k$

It is assumed that firms wish to minimise the total cost of production for any output. Let the total cost of production be C. Then,

$$C = p_1 x_1 + p_2 x_2 + \ldots + p_k x_k \tag{4.8}$$

where $p_1, p_2, \ldots p_k$ are the prices of inputs $x_1, x_2, \ldots x_k$ respectively.

Minimising (4.8) subject to (4.7) gives the first order conditions for a constraint cost minimum

$$\begin{aligned}
p_1 - \lambda \alpha_1 \alpha_0 \, x_1^{\alpha_1 - 1} x_2^{\alpha_2} x_3^{\alpha_3} \ldots x_k^{\alpha_k} &= 0 \\
p_2 - \lambda \alpha_2 \alpha_0 \, x_1^{\alpha_1} x_2^{\alpha_2 - 1} x_3^{\alpha_3} \ldots x_k^{\alpha_k} &= 0 \\
&\vdots \\
p_k - \lambda \alpha_k \alpha_0 \, x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3} \ldots x_k^{\alpha_k - 1} &= 0 \\
Q - \alpha_0 \, x_1^{\alpha_1} x_2^{\alpha_2} \ldots\ldots\ldots\ldots x_k^{\alpha_k} &= 0
\end{aligned} \tag{4.9}$$

where λ is the Lagrange multiplier associated with (eq.4.7). We now have (K + 1) equations in (K + 1) unknowns which can be solved for obtaining the values of $\lambda, x_1, x_2, \ldots x_k$.

Let us assume that electricity is the K^{th} input. Then the demand for electricity, x_k, is given by the expression

$$x_k = \beta_0 \; p_1^{\beta_1} \; p_2^{\beta_2} \; \cdots \; p_k^{\beta_k} \; Q^{\beta_{k+1}} \qquad (4.10)$$

where

β_j = parameters of $p_1 \ldots p_k$ for $j = 1, 2, \ldots k$ and

β_j parameters represent combinations of α_j, $j = 0, 1, \ldots k$

Equation (4.10) shows that demand for electricity is an exponential function of K input prices and the output. The demand model given by Eq. (4.10) forms the basis for the set of models given by equations (4.11) to (4.14) for obtaining electricity demand.

In deciding on the appropriate variables to be included in the equations for demand, a reconcillation is necessary between the needs of the hypotheses and the limits imposed by statistical estimation theory. Equations with the most desirable economic properties and explanatory ability may well have undesirable statistical properties, particularly because of multicollinearity between independent variables. When the risk of bias in parameters are felt to be particularly acute it is preferrable to take theoretically less satisfactory combinations of variables.

With the above considerations we propose the following group of four equations based on model given

by equation (4.10). The linear form of equation (4.10) is obtained by taking logarithms of both sides. The price variables chosen are:

(1) price of electricity relative to fuel price index,

(2) price of electricity relative to wage rates.

Each of these variables along with output have been included in seperate equations to minimise the effect of multicollinearity between independent variables on bias and loss of significance of parameters estimated.

$$\text{Log } D_t = \beta_0 + \beta_1 \text{ Log } Q_t + \epsilon_t \ldots \tag{4.11}$$

$$\text{Log } D_t = \beta'_0 + \beta'_1 \text{ Log } Q_t + \beta'_2 \text{ Log } \left(\frac{P}{F}\right)_t + \epsilon'_t \tag{4.12}$$

$$\text{Log } D_t = \beta^*_0 + \beta^*_1 \text{ Log } Q_t + \beta^*_2 \text{ Log } \left(\frac{P}{W}\right)_t + \epsilon^*_t \tag{4.13}$$

$$\text{Log } D_t = \beta''_0 + \beta''_1 \text{ Log } Q_t + \beta''_2 \, t + \epsilon''_t \tag{4.14}$$

where

D_t = Electricity consumption in KWH

Q_t = Index of production

P = Price of electricity (Rs.)

F = Price index of all other fuels

W = Average Wage rates (Rs.)

t = Time (Years)

ϵ_t = The disturbance term

β values are the parameters.

Model 2

Model 1 lays emphasis on relative price and output as explanatory variables. Model 2, though complementary to Model 1, lays emphasis on changes in fuel technology. Generally in studies relating to inclusion of the technological progress as a relevant independent variable, the time trend is used as a surrogate for technological progress. For the present study this surrogate variable is undesirable both from statistical and economical point of view. Since most variables used in the analysis are strongly time - trended inclusion of time as an independent variable will lead to multicollinearity and consequent statistical problem of estimation. Further time as a variable does not explain anything of the process by which technological change takes place. Instead it is felt that in many industries the most important change due to advancement in fuel technology would be reflected in the declining use of coal. Hence it is our contention that coal and coke consumption for the industry can be included as a surrogate for technological improvement.

On the other hand some of the substitution might have been due to relative price changes. That is why Model 1 and Model 2 are complementary. If Model 1 suggests that relative fuel price had little effect on electricity demand, the coal variable will reflect

influence of change in fuel technology. In case electricity demand appears to have been influenced by relative price changes, the coal variable will represent total effect of price and technological changes. Eventhough inclusion of the fuel price relative and coal variable togather may seperate out the two effects, the problem of statistical distortion due to intercorrelation between these two variables makes this approach unattractive.

The following groups of equations (4.15) to (4.19) are based on the above considerations, i.e., coal consumption is an effective surrogate for technological change. Further to test the effect of lags one period lagged coal consumption is included in equations (4.15, 4.16) The other major variable is the industrial output. An attempt has been made to study the effect of inclusion of a time trend in equation (4.16). Employment/output ratios are included in equation (4.18) to isolate the labour intensity aspect of change in technology. Investment/output ratio has been incorporated in equation (4.19) to isolate the capital intensity aspect of technological progress.

$$Z_t = \alpha_0 + \alpha_1 Q_t + \alpha_2 C_t + \alpha_3 C_{t-1} + e_t \qquad (4.15)$$

$$Z_t = \alpha'_0 + \alpha'_1 Q_t + \alpha'_2 C_t + \alpha'_3 C_{t-1} + \alpha'_4 t + e'_t \qquad (4.16)$$

$$Z_t = \alpha''_0 + \alpha''_1 Q_t + \alpha''_2 C_t + e''_t \tag{4.17}$$

$$Z_t = \alpha^*_0 + \alpha^*_1 Q_t + \alpha^*_2 C_t + \alpha^*_3 (\tfrac{N}{Q})_t + e^*_t \tag{4.18}$$

$$Z_t = \tilde{\alpha}_0 + \tilde{\alpha}_1 (I/Q)_t + \tilde{\alpha}_2 C_t + \tilde{\alpha}_3 \tilde{e}_t \tag{4.19}$$

Where

Z_t = Electricity consumed in coal equivalent units

C_t = Coal consumed in period t

C_{t-1} = Coal consumed in period t-1

N = Number of employees

Q = Index of industrial production

I = Gross fixed capital formation

α values are parameters of the equations

e_t values are disturbances

t = Time.

Model 3

This model is constructed on the hypothesis that there exists a one to one relationship between changes in output and electricity consumption. The deviation from these changes are assumed to be induced by changes in relative prices, changes in labour intensity and changes in capital intensity.

It is assumed that demand for electricity is directly related to output by the following relationships

$$D_t = \alpha_t \cdot Q_t \qquad t = 1, 2, \ldots n \qquad (4.20)$$

$$\alpha_t = f(X_{1t}, X_{2t} \ldots X_{mt}) \qquad (4.21)$$

Where

D_t = Demand for electricity in KWH in time t

α_t = Proportionality factor between output and electricity demand in period t

Q_t = Output of the industry at time t

$X_{1t}, X_{2t}, \ldots X_{mt}$ = Values of relevant independent variables such as price of different fuels, labour intensity, and capital intensity.

From equation (4.20) we get,

$$\left(\frac{D}{Q} \right)_t = \alpha_t \qquad (4.22)$$

To study the effects of relative price of different fuels, labour intensity and capital intensity on consumption of electricity per unit of output, we propose the following equations. Equation (4.23) studies the effect of labour intensity by including the $\frac{M}{Q}$ variable. The influence of capital intensity on electricity is determined by equation (4.24) by incorporating the $\frac{I}{Q}$ variable. The impact of relative price of different fuels represented by the variable $\frac{P}{F}$ is obtained from equation (4.25). Effect of price of electricity relative to wage rates on electricity demand is studied by equation (4.26) incorporating the $\frac{P}{W}$ variable.

$$\text{Log}\left(\frac{D}{Q}\right)_t = a_o + a_1 \text{Log}\left(\frac{M}{Q}\right)_t + \varepsilon_t \qquad (4.23)$$

$$\text{Log}\left(\frac{D}{Q}\right)_t = a_o' + a_1' \text{Log}\left(\frac{I}{Q}\right)_t + \varepsilon_t' \qquad (4.24)$$

$$\text{Log}\left(\frac{D}{Q}\right)_t = a_o'' + a_1'' \text{Log}\left(\frac{P}{F}\right)_t + \varepsilon_t'' \qquad (4.25)$$

$$\text{Log}\left(\frac{D}{Q}\right)_t = a_o^* + a_1^* \text{Log}\left(\frac{P}{W}\right)_t + \varepsilon_t^* \qquad (4.26)$$

where,

M = Number of employees in the industry

I = Gross fixed capital stock formation

Q = Output of the industry

P = Price of electricity

F = Fuel price index

W = Wage rates of the workers in the industry

a_o, a_1 etc. are the parameters of the equation

ε_t are the disturbance terms.

4.2.3 The Use Of Time Lags

The equations (4.11) to (4.14) and (4.15) to (4.19) presented above have been reformulated to take into account the possibility that the equilibrium level of the dependent variable will not be attained until sometime after the changes in independent variables have taken place. The following pattern of adjustment of dependent to independent variable is assumed. From equation (4.12)

of model 1, we can express the equilibrium demand D_t^* as

$$D_t^* = f\left(Q_t , \left(\frac{P}{F} \right)_t \right) \tag{4.27}$$

It should be noted that this level of demand is only reached in the long run equilibrium. Assuming a flow adjustment type of equilibrium we have

$$\frac{D_t}{D_{t-1}} = \left(\frac{D_t^*}{D_{t-1}} \right)^{\lambda} \qquad 0 < \lambda < 1 \tag{4.28}$$

where λ is the parameter of adjustment. Therefore the actual demand D_t is given by

$$D_t = D_t^{*\lambda} \cdot D_{t-1}^{(1-\lambda)} \tag{4.29}$$

Taking logarithm of both sides of equation (4.29) and substituting for D_t^* from equation (4.27) we have

$$\text{Log } D_t = \lambda \log \left[f\left(Q_t, \left(\frac{P}{F}\right)_t \right) \right] + (1 - \lambda) \text{ Log } D_{t-1} \tag{4.30}$$

Assuming that equation (4.27) is exponential the long-run elasticities can be obtained by simply dividing through by an estimate of λ from equation (4.30). It can be proved that the weights in the lag distribution are geometrically declining. For proof reader is referred to Kyock's work (207).

4.2.4 Data Used For The Study And The Estimation Procedure

For Iron and Steel, Textiles, Paper, Vehicles, Engineering Mining and Quarrying, Nonferrous Metals, Chemicals, Food industries the relevant data on the various variables for the period 1961 - 71 has been obtained from various sources. The sources of data are listed in Appendix A . The ordinary least squares technique has been used for estimation of the structural coefficients.

The results of this analysis, their interpretations, and conclusions drawn on the basis of the results are presented in Chapter VI.

CHAPTER V

STOCHASTIC TIME-SERIES ANALYSIS AND FORECASTING OF ELECTRICITY DEMAND

In this chapter we shall present the methodology for building, identifying, fitting and checking time-series models for electricity demand forecasting. The methodology discussed is appropriate for discrete (sampled-data) systems. The proposed time-series models are used to forecast the future values of peak power and energy demand for India for the period 1976 to 2000 A.D. The motivations for applying stochastic time-series analysis for electricity demand forecasting has already been presented in Chapter III.

The process generating the time-series of electricity demand is a dynamic system exhibiting changes in characteristics over time. The time-series of electricity demands (peak power and energy) are sets of observations generated sequentially in time and they constitute a statistical phenomenon that evolves in time according to probabilistic laws. For such a time series prediction of the future variations of demand can only be made based on probabilistic models. Hence it is contended that the theory of stochastic time series analysis and prediction can be applied to time-series of electricity demand.

For an extensive exposition of the theory and analysis of stochastic process references are made to Doob (145), Hannan (135), Wiener (14), Wold (136), Quenouille (149), Parzen (146), Bartlett (143), Box and Jenkins (45), Jenkin and Watts (209), Whittle (138), Robinson (211), Anderson (147), Grenanden et. al (212), Rao (213), Wald (210), Bailey (144), Bartholomew (150) and others.

In the following paragraph a brief statement of the forecasting problems when viewed from time-series analysis point of view is presented.

Let Z_t, Z_{t-1}, Z_{t-2} ... Z_{t-n} denote the chronological observations on present and past electricity demand at time t, t-1, t-2, ... t-n respectively, at discrete equispaced intervals of time. Deroting Z_t (1) as the forecast for the time period t + 1, made at origin t, for lead time 1 and Z_{t+1} as the actual demand for period t+1, our objective is to obtain a forecast function which is such that the mean square of deviations (Z_{t+1} - Z_t (1)) between the actual and forecasted values is as small as possible for each lead time 1. In addition to calculating the best forecasts we specify their accuracy so that risks associated with decisions based on these forecasts can be evaluated. The accuracy of the forecasts are expressed by calculating the probability limits on either side of the forecasts for specified levels of confidence.

5.1 ASSUMPTIONS

The following assumptions are made for formulating time-series models for electricity demand forecasting.

1. The time-series of observations Z_t, Z_{t-1} ... Z_{t-n} constitute a stationary stochastic process. This implies that the generating mechanism of the process is independent of time. The parameters and probability distributions of components of the time series are stable and time invariant. This implies that no structural changes in the parameters and process has taken place over the time span of observations.

2. If the observed process is non-stationary, then the non-stationarity is of the homogeneous category, and it is assumed that it can be converted to a stationary series by a finite linear transformation.

3. The joint probability distribution associated with m observations Z_{t_1}, Z_{t_2}, Z_{t_3} ... Z_{t_m} made sequentially at any set of equispaced intervals of time t_1, t_2 ...t_m are the same as that associated with m observations $Z_{(t_1 + k)}$, $Z_{(t_2 + k)}$ ... $Z_{(t_m + k)}$ made at times $(t_1 + k)$, $(t_2 + k)$ ... $(t_m + k)$ for all values of m and k.

5. The stochastic process is covariance stationary. This implies second order stationarity. Mathematically the second order stationarity can be expressed as

   I. for all $t \in T$ $\quad E\left[Z_t\right] =$ a constant

   II. covariance of Z_t and Z_{t+k} (where k is the lag) is a function of k only and is independent of t.

$$\text{Cov}\left[Z_t, Z_{t+k}\right] = E\left[(Z_t - \mu)(Z_{t+k} - \mu)\right]$$
$$= E\left[Z_t, Z_{t+k}\right] - E\left[Z_t\right] E\left[Z_{t+k}\right]$$
$$= \gamma_k$$

   where γ_k is referred to as autocovariance at lag k and is a function of k only.

6. The stochastic process generating the time series can be represented as a linear aggregation of random shocks and has a finite number of parameters.

7. The linear stochastic process is invertible.

5.2 STEPS IN BUILDING TIME-SERIES MODELS FOR ELECTRICITY DEMAND

We shall follow the following three basic iterative steps that are involved in building time-series models as proposed by Carlson et. al (214) and Box and Jenkins (45). The iterative approach is represented diagrammatically in figure 5.1 The steps are:

1. Identification

Using the data and any other information concerning the process, rough methods of identifying a sub-class of the general class of models are developed. The analytical tools of correlation analysis and spectral analysis are used to identify the nature and order of the process, degree of differencing and form of the models which are tentatively entertained for parsimonious (215) representation of the stochastic process. In addition the identification process also yields rough preliminary estimates of parameters.

2. Estimation

The tentatively entertained models (obtained at the identification stage) are fitted to the data and its parameters are estimated. Rough estimates of parameters obtained during identification stage are used as starting values in more refined iterative statistical methods for parameter estimation.

3. Validation

The model to be finally adopted is validated by diagnostic checks. These checks are applied with the objective of uncovering possible lack of fit and diagnosing the cause. Residuals of the fitted model are analysed to test model adequacy. If no lack of fit is

indicated, the model is finally selected. In the event of an inadequate model the iterative cycle of identification, estimation and checking is repeated till a suitable representation is found.

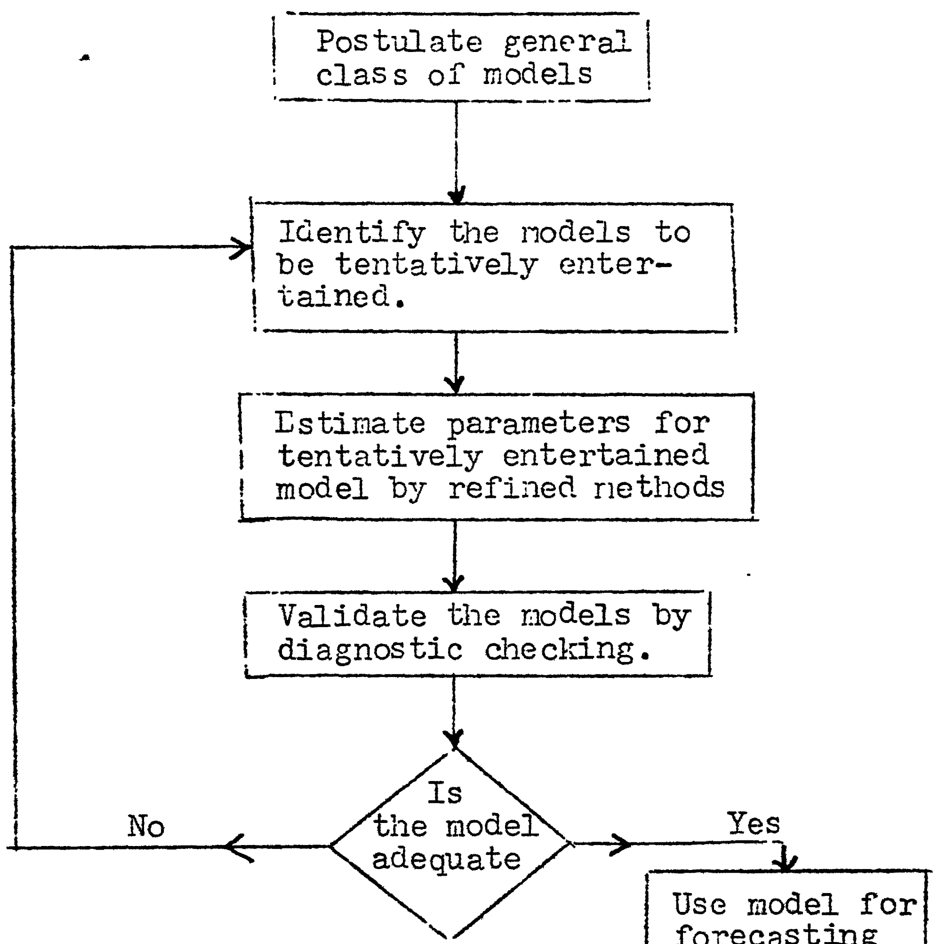


Fig. 5.1 : Stages in iterative approach to time-series model building.

We now describe in brief the procedures for identification, estimation and validation of the time-series models for electricity demand forecasting. The procedure followed is essentially the same which is suggested by Box and Jenkins (45).

5.3 IDENTIFICATION OF THE TIME-SERIES MODELS

Identification methods are procedures applied to the set of data to indicate the kind of representational model worthy of further investigation.

The electricity demand time series can generally be considered to consist of a random component, and non-random components that include trend, cyclicity and persistence. The random components refer to pure random component of the process. Trend refers to the long term behaviour of the time series and is a function of time alone. Persistence refers to linkage or relationships existing between values at a given time with earlier values and may be due to internal or external dependence. Cyclicity refers to the periodic components that repeats themselves at definite periods. The final representation of the time-series in terms of several components is given by

$$Z_t = f_{trend}(t) + f_{cycle}(t) + f_{ar}(Z_{t-1}, Z_{t-2}, \dots Z_{t-n}) + f_{ma}(\epsilon_{t-1}, \epsilon_{t-2}, \dots \epsilon_{t-l}) + \epsilon_t$$

where f_{ar} and f_{ma} represent the autoregressive and moving average components, representing internal and external correlation respectively. $_t$ is the pure random component at time t.

Refering to Box and Jenkins (45), Anderson (216), Nelson (217), the autoregressive process of order (p) is denoted by AR (p) is given by

$$\phi \ (B) \ Z_t \ = \ a_t$$

The moving average process of order q is denoted by MA (q) and is given by the relation

$$\theta \ (B) \ Z_t \ = \ a_t$$

An autoregressive integrated moving average process is denoted by ARIMA (p, d, q) and is expressed as

$$\phi \ (B) \ \nabla^d Z_t \ = \ \theta \ (B) \ a_t$$

The specific aim at the identification stage is to identify the trend, persistence and periodic components of the time series and obtain some idea ot the value of p, d, q needed for the general linear ARIMA model. The identification stage also yields initial estimates of parameters. Tentative models identified at this stage provides a starting point for application of more efficient methods.

5.4 STAGES IN THE IDENTIFICATION PROCEDURE

To identify an appropriate sub-class of the general ARIMA model (45).

$$\phi (B) \nabla^{d} Z_t = \theta_o + \theta (B) a_t$$

The following steps are followed.

5.4.1 Identification of Trend

A graphical plot of the data indicates the existence of a trend, and their characteristic. A suitable functional relationship between Z_t and t; which is generally linear, exponential or polynomial is assumed. The coefficients of the functional relationship are obtained by multiple regression analysis techniques (218). Statistical significance of the coefficients at 95% confidence level are tested by 't' values. In case the trend is found to be significant, the trend component is subtracted from the data to obtain a trend free series.

5.4.2 Identification Of Degree Of Differencing

The trend free data may be non-stationary. This series is then differenced as many times as necessary to produce stationarity there by reducing the process under study to a stationary ARMA process (45).

The order of the resultant stationary autoregressive-moving average process (ARMA) has to be identified now. The principal techniques of identifying the degree of differencing, and order of the process is by the application of certain analytical tools of identification and estimation. to be described next.

5.4.3 Analytical Procedures For Identification And Estimation Of Time-Series Models

The periodic and persistence component of the time series are identified by correlation and spectral analysis. The basic analytical tools for time-series are the autocorrelation function, partial autocorrelation function, and spectral density function.

5.4.4 Correlation Analysis Of Time-Series

For the present study the autocorrelations of the time series for different lag values have been calculated by the methods suggested by Box and Jenkins (45), Anderson (216) and Jenkins and Watts (209). The power spectra of the time series have been calculated by the methods suggested by Jenkins and Watts (209). To have a crude check whether the autocorrelation is effectively zero beyond a certain lag k, a formula given by Bartlett (218) has been used. The partial autocorrelation of the time series have been obtained by solving the Yule-Walker equations (219, 220).

5.5 IDENTIFICATION OF THE ARIMA PROCESS

Having estimated the values of the PACF, and ACF for specified lag values, the PACF and ACF are plotted graphically as a function of the lags. Referring to the procedure suggested in (45) for identification of the degree of differencing for the trend-free series, use is made of the properties of stationary time series that if none of the roots of the stationary model are close to the boundary of the unit circle the ACF quickly dies out. Tendency for the ACF not to die out quickly indicates non-stationarity. The degree of differencing that is necessary to achieve stationarity has been achieved when the ACF of the difference series dies out fairly rapidly. As suggested in (45) the time series is differenced at the most two times and the first N/4 (maximum 15 to 20) estimated autocorrelations are inspected.

Having decided the value of d the general appearance of the ACF, PACF, and the spectrum of the appropriately differenced series are studied to provide clues about the presence of periodicities, and values of p, and q, (order of AR process and MA process respectively). In doing so the characteristic behaviour of the theoretical ACF, PACF, and spectrum for AR, MA, and ARMA process described in (209, 45, 216, 217) are utilised.

The models that have been tentatively identified for the electricity demand time-series based on the information provided by the estimated ACF and PACF are presented in Chapter VI.

5.6 INITIAL ESTIMATES OF PARAMETERS

The refined iterative techniques for best estimation of parameters require initial estimates as starting points. These initial estimates of parameters are determined from the estimates of ACF and PACF obtained at the identification stage. For obtaining initial estimates of A.R. parameters the Yule-Walker (219, 220) equations are solved. For solving the Yule-Walker equations the theoretical autocorrelations are replaced by estimated autocorrelations.

Referring to Box and Jenkins (45) we write

$$\emptyset = \begin{bmatrix} \hat{\emptyset}_1 \\ \hat{\emptyset}_2 \\ \vdots \\ \hat{\theta}_p \end{bmatrix}, \quad r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix} \quad \text{and} \quad R_p = \begin{bmatrix} 1 & r_1 & r_2 & \cdots & r_{p-1} \\ r_1 & 1 & r_1 & \cdots & r_{p-2} \\ \vdots & & & & \\ r_{p-1} & r_{p-2} & & \cdots & 1 \end{bmatrix}$$

where

$\hat{\emptyset}_1, \hat{\emptyset}_2 \ldots \hat{\emptyset}_p$ are estimates of parameters to be obtained

and

$r_1, r_2 \ldots r_p$ are the sample autocorrelations.

The solution of parameters ϕ's in terms of sample autocorrelations are given by

$$\hat{\phi} = R_p^{-1} \cdot r$$

The initial estimates of moving average parameters are obtained by solving the following set of nonlinear q simultaneous equations (45).

$$r_k = \frac{\theta_k + \theta_1 \theta_{k+1} + \ldots + \theta_{q-k} \cdot \theta_q}{(1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2)} \qquad k = 1, 2, \ldots q$$

For a mixed ARMA process, Box and Jenkins (45) have suggested a computational algorithm which can be used for obtaining initial estimates of parameters of ARMA process.

The results for the initial estimation of parameters are provided in Chapter VI.

5.7 MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS

The identification process having led to a tentative formulation of the model, the efficient estimates of parameters are obtained by a refined iterative technique referred to as the maximum likelihood technique. Details of the procedure are presented in Box and Jenkins (45). An exhaustive exposition of the likelihood principle has been presented by various authorssuch as Fisher (221),

Barnard (222), Birnbaum (223). In the following paragraphs we give a brief description of the likelihood function.

Let us associate an N dimensional random variable to the sample of N observations $Z_1, Z_2 \ldots Z_n$ whose known probability distribution $p\ (Z \mid \xi)$ depends on unknown parameters ,

where,

Z = vector of observations

ξ = vector of parameters ϕ, θ and of size $p + q + 1$.

For a fixed ξ, before the data are available, $p\ (Z \mid \xi)$ will associate a density with each outcome of Z. After the data are available, we are led to contemplate the various possible values of ξ giving rise to the set of observations Z. The function for the purpose $L\ (\xi \mid Z)$ is called the likelihood function. The likelihood principle states that all the data has to tell us about parameters are contained in the likelihood function. Likelihood function is generally unimodal for moderately large and large sample of data, and can be approximated by a quadratic function near the maximum. Values of parameters maximising the likelihood function are called maximum likelihood estimates (MLE). For further details on the maximum likelihood method, the reader is referred to Rao (213), Whittle (224), Durbin (225), Bartlett (143), Barnard et. al (226), Kendall and Stewart (227), Mann and Wald (228), Hannan (135).

5.7.1 Likelihood Function For ARIMA Process

Let Z_{-d+1}, Z_{-d+2} ... Z_o, Z_1 ... Z_n denote the trend and cycle free time series with $N = n + d$ observations. Assuming that the above series is generated by an ARIMA (p, d, q) process, we generate a series w_1, w_2, ... w_n of $n = N - d$ observations,

where $w_t = \nabla^d Z_t$

Fitting ARIMA (p, d, q) to the original series (trend and cycle free) is equivalent to fitting ARMA (p, q) to the w series. We can express the residuals a_t as

$$a_t = w_t - \phi_1 w_{t-1} - \phi_2 w_{t-2} \dots - \phi_p w_{t-p} + \theta_1 a_{t-1} \dots + \theta_q a_{t-q} . \qquad t = 1, 2, \dots n$$

Normally with $d > 0$, the mean of w_t series is zero. Otherwise $\bar{w} = (\sum_{t=1}^{n} w) / n$ can be substituted for the mean of w series, and w_t's have to be replaced by $(w_t - \bar{w})$.

It is assumed that the a_t's are normally distributed. The joint probability distribution of a_1 ... a_n is given by

$$p\,(a_1 \dots a_n) \propto \sigma_a^{-n} \, e^{- (\sum a_t^2 / 2 \sigma_a^2)}$$

Referring to (45) the unconditional likelihood for the general ARIMA model with $N = n + d$ is given by

$$l(\emptyset, \theta, \sigma_a) = f(\emptyset, \theta) - n \log \sigma_a - \frac{S(\emptyset, \theta)}{2\sigma_a^2}$$

The unconditional sum of squares function $S(\emptyset, \theta)$ is given by

$$S(\emptyset, \theta) = \sum_{t=-\infty}^{n} \left[a_t \mid \emptyset, \theta, w\right]^2$$

where

$$\left[a_t \mid \emptyset, \theta, w\right] = E\left[a_t \mid \emptyset, \theta, w\right]$$

For moderately large n, $f(\emptyset, \theta)$ is insignificant and the log likelihood function is dominated by $S(\emptyset, \theta)$. Hence contours of unconditional sum of squares function in parameter space $(\emptyset, \theta)$ are very nearly contours of the likelihood function. Parameters estimated by minimising sum of squares function provide close approximation to maximum likelihood estimates.

To calculate the unconditional sum of squares the a_t's are computed recursively. A preliminary back calculation provides values for $\left[w_{-j}\right]$ for $j = 0, 1, 2, \ldots$ to start the forward recursion. The procedure described in Box and Jenkins (45) supplies us the unconditional sum of squares to any desired degree of approximation.

Referring to (45) the backward model $\emptyset(F)\, w_t = \theta(F)\, e_t$ generates the back forecasts $\left[w_{-j} \mid \emptyset, \theta, w\right]$. In practice because of the stationary

character of the AR operator, estimates of $[w_t]$ beyond some point $t = -Q$, with Q of moderate size becomes essentially equal to zero.

With sufficient approximation we write

$$w_t = \phi^{-1}(B)\,\theta(B)\,a_t$$

$$= \sum_{j=0}^{\infty} \psi_j\, a_{t-j}$$

$$\simeq \sum_{j=0}^{Q} \psi_j\, a_{t-j}$$

In general the following equations are used to generate back forecasts, and a_t values.

$\phi(F)\,[w_t] = \theta(F)\,[e_t]$ generates back forecasts

$\phi(B)\,[w_t] = \theta(B)\,[a_t]$ generates the a_t values.

5.8 MAXIMUM LIKELIHOOD EXTIMATES OF PARAMETERS BY NONLINEAR ESTIMATION ALGORITHM

Maximum likelihood estimates are closely approximated by the least squares estimates which make

$$S(\phi, \theta) = \sum_{t=-\infty}^{n} \left[a_t \mid \phi, \theta, w\right]^2 \quad \text{a minimum}.$$

In practice this is replaced by a manageable finite sum $\sum_{t=1-Q}^{n} [a_t]^2$.

Considerable simplification occurs in the minimisation of the sum of squares function with respect to β, if each $f_t(\beta)$, $(t = 1, \ldots n)$ is a linear function of the parameters β. Box and Jenkins (45) have showed that the linearity status of β is different for AR, and MA process.

However the parameters are estimated in linearising the model for a_t.

Let $\beta_o = (\beta_{1,o}, \beta_{2,o} \ldots \beta_{k,o})$ represent the guessed set of parameters. Expanding $[a_t]$ in Taylor's series we have

$$[a_t] = [a_{t,o}] - \sum_{i=1}^{k} (\beta_i - \beta_{i,o}) x_{it}$$

where

$$[a_{t,o}] = [a_t \mid w, \beta_o]$$

and

$$x_{it} = \left. \frac{\partial [a_t]}{\partial \beta_i} \right|_{\beta = \beta_o}$$

Now if X denotes the $(n + Q) \times K$ matrix x_{it}, the $(n + Q)$ equations may be expressed as

$$[a_o] = X(\beta - \beta_o) + [a]$$

The adjustment $(\beta - \beta_o)$ which minimises $S(\beta) = S(\phi, \theta) = [a]^{-1}\ [a]$ may now be obtained by linear least square, i.e. by regressing the $[a_o]$'s on to the x's. The adjusted values are substituted as new guesses and the process

pure random series (white noise). Testing goodness of fit, by scrutiny of the residuals are described in Anscombe and Tukey (231) and the methods of normal plotting by Daniel (232). Box and Jenkins (45) suggest a procedure of checking the models by over fitting.

For the purpose of the present study the model adequacies are examined by three tests. They are

1. Correlation analysis of residuals
2. χ^2 test (chi-square tests)
3. Spectral analysis of residuals.

We describe them in brief in the following paragraphs.

5.9.1 <u>Correlation Analysis</u>

Referring to Box and Jenkins (45) the residuals are expressed as

$$a_t = \hat{\theta}^{-1} (B) \hat{\phi} (B) \; w_t$$

It is possible to prove that if the model is adequate, $\hat{a}_t$ become close to white noise as length of series increase. The autocorrelations of $\hat{a}_t$ are calculated by the method described in (45). Anderson has proved that with the knowledge of true parameter values ϕ, and θ the estimated autocorrelation r_k (a_t) for $t = 1, 2, \ldots n$ and $k = 1, 2, \ldots k$ are uncorrelated and distributed

approximately normally with mean zero, and variance 1/n. Hence if the calculated autocorrelation lie within 95% confidence limits the model is considered to be adequate.

5.9.2 χ^2 Test Of Goodness Of Fit

Durbin (233) has pointed out that it is dangerous to assess the statistical significance of apparent descrepancies of estimated autocorrelations, from their theoretical values based on the standard errors which are actually appropriate for theoretical autocorrelations of the residuals. Box and Pierce (234) subsequently derived the large sample variance and covariances of all the autocorrelations of $\hat{a}_t$'s for any ARMA process. It was proved by them that use of $1/n^{\frac{1}{2}}$ as standard error for r_k $(\hat{a}_t)$ would under estimate the statistical significance of apparent departures from zero of autocorrelations at low lags but could be employed for moderately large and high lags and $n^{-\frac{1}{2}}$ should be treated as an upper bound for the standard errors of r_k $(\hat{a}_t$'s).

To circumvent the above problem, instead of considering the individual autocorrelations the first twenty or so autocorrelations of a_t's are taken as a whole to indicate model adequacy. Due to a result by Box and Pierce (234), it is possible to show that if the fitted model is appropriate

$$Q = n \sum_{k=1}^{K} r_k^2 (a_t)$$

(where $r_k (\hat{a}_t)$ are the first K autocorrelation of residuals of the ARIMA (p, d, q) process , is approximately distributed as $\chi^2_{(k-p-q)}$ where n = N - d is the number of w's fitted to the model and k - p - q = degree of freedom. Test of hypothesis of model adequacy is made by comparing the calculated Q value with the $\chi^2_{critical}$ values which are obtained from a table of percentage points of χ^2. If $Q < \chi^2_{critical}$, at the given level of significance, the model is assumed to be adequate.

5.9.3 Spectral Analysis Of The Residual Series

Jenkins and Watts (209) have proved that for a pure random series in the frequency range $0 \leq f \leq f_c$ the spectral density function is constant and is equal to the variance of the process.

The number of degrees of freedom for the spectra is given by

$$\gamma = \frac{2 N_c}{m}$$

where

$$N_c = N - \frac{m}{3}$$

and m $\frac{N}{4}$ to $\frac{N}{5}$

The sample spectral estimates are distributed about the population spectrum according to (χ^2 / ν) distribution. From tables of (χ^2 / ν) distribution the confidence limits for a specified level of confidence $(1 - \alpha)$ can be obtained, otherwise from χ^2 distribution tables $\chi^2_{\alpha/2}$, and $\chi^2_{1-\frac{\alpha}{2},\nu}$ are obtained. Dividing these values by ν gives the appropriate limits for χ^2/ν. As $\frac{G_k(f)}{G(f)}$ has a χ^2/ν distribution, the series can be considered pure random at $100 \times (1 - \alpha)$ percentage confidence level if

$$\frac{\chi^2_{\alpha/2,\nu}}{\nu} \leq \frac{G_k(f)}{G(f)} \leq \frac{\chi^2_{(1-\alpha/2),\nu}}{\nu}$$

where

$G(f)$ = spectral density of the pure random series,

$G_k(f)$ = the spectral density of a_t series.

If the spectra of the $\hat{a}_t$ series lies within the above $100\,(1 - \alpha)$ % confidence limits, then $\hat{a}_t$ series is considered to be random at the given confidence level.

Results of diagnostic checks for testing model adequacy have been presented in Chapter VI.

5.10 FORECASTING THE FUTURE VALUES OF PEAK POWER AND ENERGY DEMAND BASED ON THE TIME SERIES MODELS ADOPTED

Having selected the models of the time series of electricity demands the forecasts of future values of electricity demand, the variants of these forecasts and their probability limits for specified levels of confidence are calculated by the procedure described by Box and Jenkins (45).

CHAPTER VI

RESULTS, DISCUSSIONS AND CONCLUSIONS

In Chapter IV econometric models were developed for forecasting the future demand for peak power and energy, and for analysis of electricity demand in the industrial sector. Stochastic time-series forecasting models were formulated in Chapter V for determination of forecasts of future demand for electricity. This chapter is devoted to the presentation of the numerical results obtained by the application of the above models to actual case studies. In Sections 6.1 and 6.2 results for macro-level forecasts of peak demand and energy for India for the period 1976 - 2000 are presented. Section 6.3 deals with the results of the analysis carried out for industrial electricity demand. The industries considered are Steel, Textiles, Paper, Chemicals, Non-ferrous metals, Food, Vehicles and Mining and Quarrying. Section 6.4 presents the numerical results of forecasts of peak power and energy demand for India for the time horizon of 1976 to 2000 A.D. by the application of stochastic time-series models.

6.1 COEFFICIENT ESTIMATES OF ECONOMETRIC MODELS FORECASTING PEAK POWER AND ENERGY DEMANDS

The functional relationships presented in Chapter 4 (equations 4.3, 4.5, 4.5, 4.6) for energy and peak demand were estimated by ordinary least square technique. In the first stage all the explanatory variables were incorporated in the equations to be estimated. The simple correlation matrix indicated significant collinearity between all the independent variables, indicating the existence of multicollinearity. Because of this, it was inferred that most of the parameter estimates were highly biased, and insignificant. As all the explanatory variables are significantly time-trended, inclusion of time as an independent variable with equations was ruled out, to avoid the problem of multicollinearity. Various combination of variables for each of the simple, double log, first differenced, and first differences of logged variables form of equations were tried. The most satisfactory form of equations on the basis of both economic theory and statistical inference in its estimated form for forecasting energy and peak demand were found to be of the following.

<u>Model 1</u>

$$E_t = \underset{(7.07)}{-6183.96} + \underset{(44.97)}{200.99}\, GNP_t + \epsilon_t \qquad \bar{R}^2 = 0.9777 \quad DW = 1.876$$

Model 2

$$E_t = \underset{(10.99)}{-21336.23} + \underset{(26.22)}{399.22}\ I_t + e_t \qquad \bar{R}^2 = 0.9372 \quad DW = 2.235$$

Model 3

$$E_t = \underset{(12.67)}{-79004.7} + \underset{(16.70)}{254.58}\ P_t + e_t \qquad \bar{R}^2 = 0.8580 \quad DW = 1.579$$

Model 4

$$\text{Log } E_t = \underset{(25.974)}{3.344} + \underset{(45.904)}{1.301}\ \text{Log } GNP_t + e_t \qquad \bar{R}^2 = 0.9786 \quad DW = 1.868$$

Model 5

$$\text{Log } E_t = \underset{(83.857)}{-24.472} + \underset{(115.09)}{5.640}\ \text{Log } P_t + e_t \qquad \bar{R}^2 = 0.9965 \quad DW = 2.02$$

Model 6

$$\text{Log } E_t = \underset{(5.69)}{-1.709} + \underset{(36.255)}{2.397}\ \text{Log } I_t + e_t \qquad \bar{R}^2 = 0.9662 \quad DW = 2.65$$

where

E_t = Electric energy demanded in period t,

GNP_t = Gross national product (indices) in period t,

DW = Durbin - Watson statistic.

Models Adopted For Peak Demand

Model 1

$$PD_t = \underset{(6.361)}{-638.681} + \underset{(60.923)}{31.299}\ GNP_t + e_t \qquad \bar{R}^2 = 0.9878 \quad DW = 1.932$$

Model 2

$$PD_t = \underset{(17.180)}{-3130.17} + \underset{(44.562)}{63.249}\ I_t + \epsilon_t \qquad \bar{R}^2 = 0.9773 \quad DW = 2.189$$

Model 3

$$PD_t = \underset{(14.57)}{-12209.00} + \underset{(19.633)}{40.225}\ P_t + \epsilon_t \qquad \bar{R}^2 = 0.8931 \quad DW = 1.965$$

Model 4

$$\text{Log } PD_t = \underset{(17.23)}{2.642} + \underset{(32.68)}{1.102}\ \text{Log } GNP_t + \epsilon_t \qquad \bar{R}^2 = 0.9586 \quad DW = 2.054$$

Model 5

$$\text{Log } PD_t = \underset{(46.29)}{-21.09} + \underset{(62.81)}{4.807}\ \text{Log } P_t + \epsilon_t \qquad \bar{R}^2 = 0.9885 \quad DW = 2.73$$

Model 6

$$\text{Log } PD_t = \underset{(16.216)}{-1.857} + \underset{(82.39)}{2.079}\ \text{Log } I_t + \epsilon_t \qquad \bar{R}^2 = 0.9933 \quad DW = 1.793$$

where

PD_t = Peak demand (in MW) in period t.

For all the above equations the figures in the parenthesis under the coefficients are their corresponding 't' values. $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom. DW is the Durbin-Watson statistic.

The coefficient of determination for each equation is significantly high; there by indicating that

these structural equations are satisfactory and are adequate representation of electricity demand in the past. Gross national product, population, and industrial output are appropriate explanatory variables for representing past demand for electricity. Further the 't' values for the coefficients estimated are quite high indicating that the explanatory variables are significant in explaining the causality between the dependent and independent variables. The values of the D.W. statistic signify that the hypothesis of no autocorrelation in error terms cannot be rejected at the 95% confidence level.

The coefficients for the linear functional form of equations can be interpreted in the following manner. A unit increase in the index of GNP using 1961-62 as the base year will raise the demand for electricity by 201 million KWH. From the relationship linking industrial output and electricity energy demand we find that a rise in industrial output leading to a unit rise in the index of industrial output will generate demand for an additional 399.92 million KWH of electricity energy. Further, from the estimated relationship for population, the coefficient suggest that a million increase in population will lead to an increase of demand for energy by 254.58 million KWH. Comparing the $\bar{R}^2$ values of the simple linear relationships per energy demand it is observed that population explains

about 85.8% of the total variance of demand for electricity energy where as the corresponding figures for GNP and industrial output are 97.77% and 93.72% respectively. This leads us to suggest that amongst the three explanatory variables the GNP is a more appropriate causal variable, explaining the largest variance in electricity consumption followed by industrial output and population in that order.

From the results we observe that electricity requirements for a unit increase in industrial output is almost double that of the requirements for an unit rise in GNP. This result is as expected. We provide the following explanation for this phenomenon. The available data on electricity consumption by various sectors of the economy indicate that, industrial sector accounts for about 75% of the total consumption. Hence a unit rise in index of output may lead to such heavy requirements. Further the gross national product of country comprises of output from all sectors of the economy. All the sectors are not such disproportionately heavy users of electricity as the industrial sector, compared to their output. For example the transportation sector mostly relies upon coal and oil as its major source of energy. Despite the growing needs of the agricultural sector for energisation of tube wells and mechanisation, this sector still has a minor share in the total consumption of electricity. In addition the

contribution of industrial output to GNP is only about 60% meaning thereby that a unit increase in GNP has received about 0.6 of its value from the industrial sector. From these considerations it is clear that a unit rise in GNP will require lesser amount of electricity energy as compared to the requirements for a unit rise in industrial output.

The structural coefficients of the log linear form of equation represent the elasticities of demand corresponding to the variable under discussion. The income elasticity of demand for energy as well as the population elasticity and industrial output elasticity are greater than unity as well as for population and industrial output. One percent rise in the GNP gives rise to 1.3% rise in the demand for electricity energy. Similarly 1% rise in industrial output and 1% increase in population gives rise 2.4% and 5.65% increase in demand for electricity energy respectively. From the log linear form of equations also we see that the requirements of electricity energy for rise in industrial output is almost double that of the requirements for one percent rise in GNP. As in the simple linear form of equations (equations 4.3 and 4.4 of chapter 4) the variables have maintained their relative positions of explanatory ability.

The results of the models for peak demand can be interpreted in an exactly similar manner. Here also

the results indicate that GNP is the most appropriate explanatory variable explaining the largest variance in electricity consumption followed by industrial output and population. A unit change in index of GNP, industrial output and population gives rise to an additional peak demand of 31.29, 63.24, and 40.22 MW respectively. Similarly one percent increase in GNP, industrial output and population give rise to a 1.10%, 2.08%, 4.80% increase in peak power demand respectively.

The knowledge of the elasticity coefficients are useful for policy decisions. For example, when plans are formulated with an objective to increase the GNP by a certain amount, or the industrial output by a certain amount, suitable investments have to be made in the power sector to meet this additional demand as a consequence of increase in output. Similarly appropriate policy measures have to be taken in terms of determining the optimal mix, location, and time phasing of these additional capacity that has to be added to meet the demand.

6.2 FORECASTS OF FUTURE DEMANDS FOR PEAK POWER AND ENERGY FOR INDIA

The forecasts of peak power and energy demand have been obtained for India for the years 1976 to 2000, on the basis of the estimated relationships. The future

values of GNP, population, and industrial output have been estimated by applying planned and anticipated growth rates. The planned and anticipated growth rates have been taken from a publication of the Planning Commission, Government of India (235). Substituting these values of the independent variables in the estimated demand equation we obtain point value of predictions. Interval predictions for 95% level of confidence, (for the individual values of forecasts) have also been presented. Numerical results have been presented for both peak power and energy demand for twentyfive years in future for the various growth rates. Growth rates considered are - 5%, 6% and 7% for GNP; 8%, 9% and 10% for industrial output; 1.5%, 2.0% and 2.25% for population. The anticipated and planned growth rates according to the Planning Commission (235) for GNP, industrial output and population are 5%, 8% and 1.5% respectively.

Tables 1 to 3 present results for energy demand and Tables 4 to 6 present results for peak demand forecasts obtained from the simple linear form of equations (equations 4.3 and 4.4). Tables 7 to 9 present results for energy demand and Tables 10 to 12 present results for peak demand forecasts obtained from log linear form of equations (equations 4.5 and 4.6).

TABLE 1

Forecast of Energy Consumption from Gross National Product (GNP) Projections.

Level of Confidence - 95% All forecast values are to be multiplied by 10^5

YEAR	ANNUAL GROWTH RATE OF G.N.P 5%			6%			7%		
	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of forecasts
1976	0.97862	1.01457	0.94266	0.98853	1.02489	0.95216	0.99843	1.03522	0.96165
1977	1.03064	1.06877	0.99251	1.05155	1.09056	1.01254	1.07265	1.11256	1.03275
1978	1.08526	1.12570	1.04483	1.11835	1.16019	1.07652	1.15207	1.19534	1.10880
1979	1.14262	1.18548	1.09975	1.11891	1.23401	1.14432	1.23704	1.28394	1.19015
1980	1.20284	1.24827	1.15741	1.26422	1.31228	1.21616	1.32797	1.37876	1.27717
1981	1.26608	1.31422	1.21794	1.34379	1.39527	1.29231	1.42525	1.48025	1.37025
1982	1.33247	1.38347	1.28148	1.42813	1.48325	1.37300	1.52935	1.58886	1.46984
1983	1.40219	1.45619	1.34819	1.51752	1.57652	1.45853	1.64073	1.70509	1.57638
1984	1.47539	1.53256	1.41822	1.61229	1.67540	1.54917	1.75991	1.82947	1.69036
1985	1.55225	1.61276	1.49175	1.71273	1.78023	1.64524	1.75991	1.96257	1.81230
1986	1.63296	1.69697	1.56894	1.81921	1.89135	1.74706	1.88744	2.10500	1.94277
1987	1.71770	1.78541	1.64990	1.93207	2.00916	1.85498	2.02388	2.25741	2.08236
1988	1.80667	1.87827	1.73507	2.05171	2.13404	1.96937	2.16989	2.42050	2.23171
1989	1.90010	1.97579	1.82441	2.17852	2.26642	2.09061	2.32611	2.59502	2.39151
1990	1.99820	2.07818	1.91821	2.31294	2.40676	2.21912	2.49326	2.78176	2.56248
1991	2.10120	2.18571	2.01669	2.45543	2.55552	2.35534	2.67212	2.98159	2.74541
1992	2.20935	2.29861	2.12009	2.60646	2.71321	2.49972	2.86350	3.19541	2.94114
1993	2.32291	2.41717	2.22866	2.76656	2.88037	2.65275	3.06827	3.42420	3.15056
1994	2.44215	2.54165	2.34264	2.93627	3.05757	2.81497	3.28738	3.66901	3.37464
1995	2.56735	2.67237	2.46233	3.11615	3.24540	2.98690	3.52183	3.93097	3.61440
1996	2.69881	2.80963	2.58799	3.30683	3.44451	3.16916	3.77263	4.21127	3.87093
1997	2.83684	2.95375	2.71993	3.50895	3.65557	3.36234	4.32831	4.51119	4.14542
1998	2.98177	3.10509	2.85846	3.72320	3.87930	3.56711	4.63562	4.83212	4.43912
1999	3.13396	3.26399	3.00392	3.95030	4.11645	3.78416	4.96444	5.17551	4.75337
2000	3.29375	3.43084	3.15665	4.19103	4.36784	4.01423	5.31628	5.55294	5.08961

TABLE 2

Forecast of Energy Consumption from Industrial Output Projections

Level of Confidence - 95% All forecasts values are to be multiplied by 10^5

YEAR	ANNUAL GROWTH RATE OF INDUSTRIAL OUTPUT 8%			9%			10%		
	Point fore-casts	Upper limits of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of forecasts
1976	0.90961	0.96620	0.85303	0.92001	0.97734	0.86269	0.93041	0.98847	0.87235
1977	0.99945	1.06245	0.93646	1.02202	1.08664	0.95739	1.04479	1.11105	0.97852
1978	1.09648	1.16649	1.02647	1.13320	1.20589	1.06051	1.17060	1.24602	1.09518
1979	1.20127	1.27893	1.12360	1.25439	1.33596	1.17282	1.30900	1.39460	1.22340
1980	1.31444	1.40043	1.22844	1.38649	1.47782	1.29516	1.46124	1.55812	1.36436
1981	1.43666	1.53171	1.34161	1.53048	1.63251	1.42844	1.62870	1.73806	1.51933
1982	1.56866	1.67351	1.46378	1.68742	1.80118	1.57367	1.81290	1.93606	1.68975
1983	1.71122	1.82676	1.59569	1.85849	1.98507	1.73191	2.01553	2.15392	1.87714
1984	1.86519	1.99228	1.73814	2.04496	2.18556	1.90436	2.23842	2.39360	2.08324
1985	2.03148	2.17106	1.89189	2.24621	2.40413	2.09229	2.48360	2.65729	2.30991
1986	2.21106	2.36418	2.05795	2.46975	2.64240	2.29711	2.75330	2.94739	2.55921
1987	2.40502	2.57727	2.23726	2.71123	2.90214	2.52033	3.04996	3.26052	2.83341
1988	2.61449	2.79808	2.43090	2.97445	3.18528	2.76361	3.37629	3.61759	3.13500
1989	2.84072	3.04143	2.64001	3.26135	3.49393	3.02877	3.73526	4.00379	3.46673
1990	3.08505	3.30426	2.86583	3.57408	3.83037	3.31778	4.13012	4.42862	3.83162
1991	3.34892	3.58814	3.10970	3.91494	4.19711	3.63278	4.56447	4.89596	4.23298
1992	3.63390	3.89474	3.37307	4.28649	4.59687	3.97612	5.04226	5.41005	4.67446
1993	3.94168	4.22588	3.65749	4.69148	5.03262	4.35034	5.56782	5.97556	5.16008
1994	4.27409	4.58352	3.96465	5.13292	5.50760	4.75823	6.14594	6.59763	5.69424
1995	4.63308	4.96979	4.29638	5.61408	6.20534	5.20282	6.78187	7.28192	6.28181
1996	5.02080	5.38696	4.65463	6.13855	6.58962	5.68742	7.48139	8.03465	6.92813
1997	5.43953	5.83752	5.04154	6.71022	7.20483	6.21562	8.25087	8.86266	7.63907
1998	5.89176	6.32413	5.45939	7.33335	7.87535	6.79135	9.09729	9.77348	8.42110
1999	6.38017	6.84968	5.91066	8.01255	8.60622	7.41888	10.02836	10.77539	9.28132
2000	6.90766	7.41728	6.39803	8.75288	9.40287	8.10290	11.05253	11.87749	10.22756

TABLE 3

Forecasts of Energy Demand from Population Projections

Level of Confidence - 95%. All forecast values are to be multiplied by 10^5

YEAR	ANNUAL GROWTH RATE OF POPULATION 1.5% Point fore-casts	1.5% Upper limit of fore-casts	1.5% Lower limit of fore-casts	2.0% Point fore-casts	2.0% Upper limit of fore-casts	2.0% Lower limit of fore-casts	2.25% Point fore-casts	2.25% Upper limit of fore-casts	2.25% Lower limit of forecasts
1976	0.35736	0.43070	0.28402	0.35927	0.43345	0.28510	0.36023	0.43482	0.28564
1977	0.36319	0.43907	0.28730	0.36708	0.44468	0.28948	0.36903	0.44749	0.29057
1978	0.36910	0.44759	0.29061	0.37504	0.45616	0.29392	0.37803	0.46049	0.29558
1979	0.37510	0.45625	0.29395	0.38316	0.46790	0.29842	0.38724	0.47380	0.30067
1980	0.38119	0.46506	0.29733	0.39145	0.47990	0.30299	0.39665	0.48745	0.30585
1981	0.38738	0.47400	0.30075	0.39990	0.49217	0.30763	0.40627	0.50144	0.31111
1982	0.39365	0.48310	0.30420	0.40852	0.50470	0.31233	0.41611	0.51576	0.31647
1983	0.40027	0.49235	0.30769	0.41731	0.51750	0.31711	0.42617	0.53043	0.32192
1984	0.40649	0.50175	0.31123	0.42627	0.53057	0.32197	0.43646	0.54545	0.32747
1985	0.41305	0.51130	0.31480	0.43542	0.54393	0.32691	0.44698	0.56082	0.33314
1986	0.41971	0.52101	0.31842	0.44475	0.55756	0.33194	0.45773	0.57656	0.33891
1987	0.42647	0.53087	0.32208	0.45426	0.57148	0.33705	0.46873	0.59267	0.34479
1988	0.43334	0.54088	0.32579	0.46397	0.58570	0.34225	0.47998	0.60916	0.35079
1989	0.44030	0.55106	0.32954	0.47387	0.60020	0.34754	0.49147	0.62603	0.35692
1990	0.44737	0.56140	0.33335	0.48397	0.61502	0.35292	0.50323	0.64330	0.36317
1991	0.45455	0.57190	0.33720	0.49427	0.63013	0.35840	0.51525	0.66096	0.36954
1992	0.46183	0.58256	0.34110	0.50477	0.64556	0.36399	0.52754	0.67903	0.37605
1993	0.46923	0.59340	0.34506	0.51549	0.66131	0.36967	0.54011	0.69752	0.38270
1994	0.47673	0.60440	0.34906	0.52642	0.67738	0.37546	0.55296	0.71644	0.38949
1995	0.48435	0.61557	0.35312	0.53757	0.69378	0.38136	0.56610	0.73579	0.39642
1996	0.49208	0.62692	0.35724	0.54894	0.71052	0.38737	0.57954	0.75558	0.40350
1997	0.49992	0.63844	0.36141	0.56054	0.72760	0.39349	0.59328	0.77583	0.41073
1998	0.50789	0.65014	0.36564	0.57237	0.74503	0.39972	0.60732	0.79653	0.41811
1999	0.51597	0.66202	0.36993	0.58444	0.76281	0.40608	0.62169	0.81772	0.42565
2000	0.52418	0.67408	0.37427	0.59675	0.78095	0.41255	0.63637	0.83938	0.43336

TABLE 4.

Forecast of Peak Demand from Gross National Product (GNP) Projections

Level of Confidence - 95%   All forecast values are to be multiplied by 10^5

YEAR	ANNUAL GROWTH RATE OF G.N.P 5% Point fore-casts	5% Upper limit of fore-casts	5% Lower limit of fore-casts	6% Point fore-casts	6% Upper limit of fore-casts	6% Lower limit of fore-casts	7% Point fore-casts	7% Upper limit of fore-casts	7% Lower limit of fore-casts
1976	0.15563	0.15976	0.15149	0.15717	0.16135	0.15299	0.15871	0.16294	0.15449
1977	0.16373	0.16811	0.15935	0.16698	0.17147	0.16250	0.17027	0.17486	0.16568
1978	0.17223	0.17688	0.16759	0.17739	0.18220	0.17258	0.18264	0.18761	0.17766
1979	0.18117	0.18609	0.17624	0.18841	0.19357	0.18326	0.19587	0.20126	0.19048
1980	0.19054	0.19577	0.18532	0.20010	0.20563	0.19458	0.21003	0.21587	0.20419
1981	0.20039	0.20592	0.19486	0.21249	0.21841	0.20657	0.22518	0.23150	0.21886
1982	0.21073	0.21659	0.20487	0.22562	0.23196	0.21992	0.24139	0.24823	0.23455
1983	0.22159	0.22779	0.21538	0.23955	0.24633	0.23276	0.25873	0.26613	0.25133
1984	0.23298	0.23956	0.22641	0.25430	0.26156	0.24705	0.27729	0.28523	0.26930
1985	0.24495	0.25191	0.23800	0.26994	0.27770	0.26219	0.29715	0.30578	0.28851
1986	0.25752	0.26488	0.25016	0.28652	0.29482	0.27823	0.31840	0.32772	0.30907
1987	0.27072	0.27850	0.26293	0.30410	0.31296	0.29524	0.34113	0.35119	0.33107
1988	0.28457	0.29280	0.27634	0.32273	0.33219	0.31326	0.36546	0.37312	0.35461
1989	0.29912	0.30782	0.29042	0.34247	0.35258	0.33237	0.39149	0.40318	0.37979
1990	0.31440	0.32359	0.30520	0.36341	0.37419	0.35262	0.41934	0.43194	0.40674
1991	0.33043	0.34105	0.32072	0.38559	0.39710	0.37409	0.44914	0.46271	0.43557
1992	0.34728	0.35754	0.33702	0.40911	0.42138	0.39684	0.48103	0.49564	0.46641
1993	0.36496	0.37579	0.35413	0.43404	0.44713	0.42096	0.51515	0.53087	0.49942
1994	0.38353	0.39496	0.37209	0.46047	0.47441	0.44653	0.55165	0.56857	0.53473
1995	0.40302	0.41509	0.39095	0.48848	0.50334	0.47363	0.59072	0.60891	0.57252
1996	0.42349	0.43623	0.41076	0.51817	0.53400	0.50235	0.63251	0.65207	0.61295
1997	0.44499	0.45843	0.43155	0.54965	0.56650	0.53280	0.67724	0.69826	0.65622
1998	0.46756	0.48173	0.45338	0.58301	0.60095	0.56507	0.72509	0.74682	0.70250
1999	0.49125	0.50620	0.47631	0.61837	0.63747	0.59928	0.77629	0.80056	0.75203
2000	0.51614	0.53190	0.50038	0.65586	0.67618	0.63554	0.83108	0.85714	0.80503

Forecast of Peak Demand from Industrial Output Projections

Level of Confidence - 95% All forecast values are to be multiplied by 10^5

YEAR	ANNUAL GROWTH RATE OF INDUSTRIAL OUTPUT								
	8%			9%			10%		
	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts
1976	0.14657	0.15183	0.14130	0.14821	0.15355	0.14288	0.14986	0.15526	0.14445
1977	0.16077	0.16644	0.15491	0.16434	0.17036	0.15833	0.16794	0.17411	0.16178
1978	0.17612	0.18264	0.16960	0.18193	0.18869	0.17516	0.18784	0.19486	0.18082
1979	0.19269	0.19992	0.18546	0.20109	0.20869	0.19350	0.20735	0.21770	0.20176
1980	0.21059	0.21860	0.20259	0.22199	0.23049	0.21348	0.23381	0.24283	0.22479
1981	0.22992	0.23877	0.22907	0.24476	0.25426	0.23526	0.26029	0.27047	0.25011
1982	0.25080	0.26056	0.24103	0.26958	0.28017	0.25899	0.28942	0.30089	0.27796
1983	0.27334	0.28410	0.26259	0.29664	0.30842	0.28485	0.32147	0.33435	0.30859
1984	0.29769	0.30952	0.28587	0.32613	0.33921	0.31304	0.35672	0.37117	0.34228
1985	0.32399	0.33699	0.31100	0.35827	0.37278	0.34376	0.39550	0.41167	0.37933
1986	0.35240	0.36665	0.33814	0.39331	0.40938	0.37724	0.43815	0.45622	0.42008
1987	0.38307	0.39868	0.36745	0.43150	0.44927	0.41373	0.48507	0.50523	0.46491
1988	0.41620	0.43329	0.39911	0.47313	0.49275	0.45350	0.53668	0.55914	0.51422
1989	0.45198	0.47066	0.43329	0.51850	0.54015	0.49685	0.59345	0.61845	0.56846
1990	0.49062	0.51102	0.47021	0.56796	0.59182	0.54410	0.65590	0.68369	0.62812
1991	0.53235	0.55462	0.51008	0.62187	0.64813	0.59561	0.72459	0.75545	0.69374
1992	0.57742	0.60170	0.55314	0.68063	0.70952	0.65174	0.80016	0.83439	0.76592
1993	0.62610	0.65255	0.59965	0.74468	0.77643	0.71293	0.88328	0.92123	0.84532
1994	0.67867	0.70747	0.64987	0.81450	0.84937	0.77962	0.97471	1.01675	0.93266
1995	0.73545	0.76679	0.70410	0.89059	0.92887	0.85231	1.07528	1.12183	1.02874
1996	0.79676	0.83085	0.76268	0.97354	1.01553	0.93155	1.18591	1.23741	1.13442
1997	0.86299	0.90003	0.82594	1.06395	1.10999	1.01791	1.30761	1.36456	1.25066
1998	0.93451	0.97476	0.89426	1.16250	1.21295	1.11205	1.44147	1.50442	1.37853
1999	1.01175	1.05546	0.96805	1.26992	1.32518	1.21466	1.58873	1.65826	1.51919
2000	1.09518	1.14261	1.04774	1.38701	1.44751	1.32650	1.75070	1.82749	1.67391

TABLE 6

Forecasts of Peak Demand from Population Projections

Level of Confidence - 95% All forecast values are to be multiplied by 10^5

YEAR	Annual growth rate of Population								
	1.5%			2.0%			2.25%		
	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of forecasts
1976	0.12492	0.13478	0.11506	0.12614	0.13611	0.11617	0.12675	0.13677	0.11672
1977	0.12863	0.13883	0.11843	0.13110	0.14153	0.12067	0.13235	0.14289	0.12180
1978	0.13239	0.14294	0.12184	0.13617	0.14707	0.12526	0.13807	0.14915	0.12699
1979	0.13620	0.14711	0.12530	0.14133	0.15272	0.12994	0.14392	0.15556	0.13229
1980	0.14008	0.15135	0.12881	0.14660	0.15849	0.13471	0.14991	0.16211	0.13770
1981	0.14401	0.15566	0.13237	0.15197	0.16438	0.13957	0.15603	0.16882	0.14324
1982	0.14800	0.16003	0.13598	0.15746	0.17038	0.14453	0.16229	0.17568	0.14889
1983	0.15205	0.16447	0.13964	0.16305	0.17651	0.14958	0.16869	0.18270	0.15467
1984	0.15617	0.16897	0.14336	0.16875	0.18277	0.15473	0.17523	0.18988	0.16058
1985	0.16034	0.17355	0.14713	0.17457	0.18915	0.15998	0.18192	0.19722	0.16662
1986	0.16458	0.17819	0.15096	0.18050	0.19566	0.16534	0.18876	0.20473	0.17279
1987	0.16888	0.18294	0.15484	0.18655	0.20231	0.17079	0.19575	0.21241	0.17909
1988	0.17324	0.18770	0.15879	0.19272	0.20909	0.17636	0.20290	0.22027	0.18554
1989	0.17767	0.19256	0.16278	0.19902	0.21600	0.18204	0.21022	0.22830	0.19213
1990	0.18217	0.19750	0.16684	0.20544	0.22306	0.18783	0.21769	0.23652	0.19887
1991	0.18673	0.20251	0.17096	0.21199	0.23026	0.19373	0.22534	0.24492	0.20575
1992	0.19136	0.20759	0.17514	0.21868	0.23760	0.19975	0.23316	0.25352	0.21279
1993	0.19607	0.21276	0.17938	0.22549	0.24509	0.20589	0.24115	0.26231	0.21999
1994	0.20084	0.21800	0.18368	0.23244	0.25273	0.21215	0.24932	0.27130	0.22735
1995	0.20568	0.22332	0.18804	0.23953	0.26053	0.21854	0.25768	0.28049	0.23487
1996	0.21060	0.22872	0.19247	0.24677	0.26848	0.22505	0.26622	0.28989	0.24256
1997	0.21559	0.23421	0.19697	0.25414	0.27660	0.23169	0.27496	0.29950	0.25042
1998	0.22066	0.23978	0.20153	0.26167	0.28488	0.23846	0.28389	0.30933	0.25846
1999	0.22580	0.24543	0.20616	0.26934	0.29332	0.24537	0.29303	0.31935	0.26668
2000	0.23101	0.25117	0.21086	0.27717	0.20193	0.25241	0.30273	0.32966	0.27508

TABLE 7

Forecasts of Energy Demand From Gross National Product (GNP) Projections
(Log linear form of equations)

Level of Confidence - 95%   All forecast values are log transformed values (to base e)

YEAR	5%			6%			7%		
	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of forecasts
1976	11.4768	11.5977	11.3558	11.4891	11.6105	11.3677	11.5013	11.2232	11.2795
1977	11.5403	11.6636	11.4169	11.5650	11.6892	11.4407	11.5894	11.7146	11.4642
1978	11.6038	11.7295	11.4780	11.6408	11.7679	11.5136	11.6774	11.8060	11.5489
1979	11.6673	11.7954	11.5391	11.7166	11.8467	11.5866	11.7655	11.8974	11.6336
1980	11.7308	11.8613	11.6002	11.7924	11.9254	11.6595	11.8535	11.9889	11.7182
1981	11.7943	11.9273	11.6613	11.8683	12.0042	11.7324	11.9416	12.0803	11.8029
1982	11.8578	11.9932	11.7223	11.9441	12.0829	11.8053	12.0296	12.1718	11.8875
1983	11.9213	12.0592	11.7833	12.0199	12.1617	11.8782	12.1177	12.2633	11.9721
1984	11.9848	12.1252	11.8444	12.0958	12.2405	11.9510	12.2005	12.3548	12.0567
1985	12.0482	12.1911	11.9054	12.1716	12.3193	12.0239	12.2938	12.4464	12.1412
1986	12.1117	12.2571	11.9664	12.2474	12.3982	12.0967	12.3819	12.5379	12.2258
1987	12.1752	12.3231	12.0274	12.3233	12.4770	12.1695	12.4699	12.6295	12.3103
1988	12.2387	12.3891	12.0883	12.3991	12.5559	12.2423	12.5580	12.7211	12.3948
1989	12.3022	12.4551	12.1493	12.4749	12.6347	12.3151	12.6460	12.8127	12.4793
1990	12.3657	12.5212	12.2103	12.5508	12.7136	12.3879	12.7341	12.9043	12.5638
1991	12.4292	12.5875	12.2712	12.6266	12.7925	12.4607	12.8221	12.9959	12.6483
1992	12.4927	12.6532	12.3322	12.7024	12.8714	12.5334	12.9102	13.0876	12.7327
1993	12.5562	12.7193	12.3931	12.7783	12.9503	12.6062	12.9982	13.1792	12.8172
1994	12.6197	12.7853	12.4541	12.8541	13.0292	12.6790	13.0863	13.2709	12.9016
1995	12.6832	12.8514	12.5150	12.9299	13.1081	12.7517	13.1743	13.3625	12.9861
1996	12.7467	12.9175	12.5759	13.0057	13.1871	12.8244	13.2624	13.4542	13.0705
1997	12.8102	12.9835	12.6368	13.0816	13.2660	12.8972	13.3504	13.5459	13.1549
1998	12.8737	13.0496	12.6978	13.1574	13.3449	12.9699	13.4385	13.6376	13.2394
1999	12.9372	13.1157	12.7587	13.2332	13.4239	13.0462	13.5625	13.7293	13.3238
2000	13.0007	13.1818	12.8196	13.3091	13.5028	13.1153	13.6146	13.8210	13.4082

TAPLE 8

Forecasts of Energy Demand From Industrial Output Projections
(Log linear form of equations)

Level of Confidence - 95%   All forecast values are log transformed values (to base e)

YEAR	Annual growth rate of industrial output: 8%			9%			10%		
	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of fore-casts
1976	11.8056	11.9743	11.3368	11.8277	12.0205	12.6787	11.8496	11.9975	11.6578
1977	11.9901	12.1679	11.8123	12.0343	12.2602	11.8959	12.0780	12.2143	11.8543
1978	12.1746	12.3616	11.9878	12.2409	12.5002	12.1129	12.3065	12.4312	12.0505
1979	12.3591	12.5554	12.1628	12.4474	12.7402	12.3298	12.5350	12.6432	12.2467
1980	12.5436	12.7492	12.3379	12.6540	12.9804	12.5466	12.7635	12.8653	12.4427
1981	12.7281	12.9432	12.5130	12.8606	13.2207	12.7633	12.9920	13.0825	12.6387
1982	12.9126	13.1372	12.6880	13.0672	13.4610	12.9799	13.2205	13.2998	12.8346
1983	13.0971	13.3312	12.8629	13.2738	13.7015	13.1965	13.4490	13.5172	13.0305
1984	13.2816	13.5253	13.0378	13.4804	13.9420	13.4130	13.6775	13.7246	13.2263
1985	13.4661	13.7195	13.2127	13.6870	14.1825	13.6294	13.9059	13.9520	13.4220
1986	13.6506	13.9136	13.3875	13.8936	14.4231	13.8458	14.1344	14.1695	13.6177
1987	13.8351	14.1079	13.5623	14.1002	14.6637	14.0622	14.3629	14.3870	13.8134
1988	14.0196	14.3021	13.7370	14.3068	14.9043	14.2785	14.5914	14.6046	14.0090
1989	14.2041	14.4964	13.9811	14.5134	15.1450	14.4984	14.8199	14.8222	14.2046
1990	14.3886	14.6907	14.0864	14.7200	15.3857	14.7110	15.0484	15.0398	14.4002
1991	14.5731	14.8850	14.2611	14.9266	15.6265	14.9273	15.2769	15.2574	14.5957
1992	14.7576	15.0794	14.4357	15.1332	15.8672	15.7435	15.5054	15.4751	14.7913
1993	14.9420	15.2737	14.6104	15.3398	16.1080	15.3597	15.7338	15.6927	14.9868
1994	15.1265	15.4681	14.7850	15.5464	16.3488	15.5758	15.9623	15.9104	15.1843
1995	15.3110	15.6625	14.9596	15.7529	16.5896	15.7920	16.1908	16.1282	15.3777
1996	15.4955	15.8569	15.1342	15.9595	16.8301	16.0081	16.4193	16.3459	15.5732
1997	15.6800	16.0513	15.3088	16.1661	17.0713	16.2243	16.6478	16.5636	15.7689
1998	15.8645	16.2458	15.4833	16.3727	17.3122	16.4403	16.8763	16.7814	15.9641
1999	16.0490	16.4402	15.6579	16.5793	17.5530	16.6565	17.1048	16.9991	16.1595
2000	16.2335	16.6347	15.8324	16.7859	17.7936	16.8726	17.3329	17.2169	16.3549

TABLE 9

Forecasts of Energy Demand From Population Projections
(log linear form of equations)

Level of Confidence - 95% All forecast values are log transformed values (to base e)

YEAR	Annual growth rate of population. 1.5% Point forecasts	1.5% Upper limit of forecasts	1.5% Lower limit of forecasts	2.0% Point forecasts	2.0% Upper limit of forecasts	2.0% Lower limit of forecasts	2.25% Point forecasts	2.25% Upper limit of forecasts	2.25% Lower limit of forecasts
1976	11.4921	11.5445	11.4398	11.5024	11.5551	11.4496	11.5075	11.5604	11.4545
1977	11.5231	11.5767	11.4695	11.5435	11.5980	11.4891	11.5537	11.6086	11.4988
1978	11.5541	11.6090	11.4991	11.5847	11.6409	11.5285	11.6000	11.6568	11.5431
1979	11.5850	11.6412	11.5288	11.6259	11.6839	11.5679	11.6463	11.7051	11.5874
1980	11.6160	11.6735	11.5584	11.6371	11.7268	11.6074	11.6925	11.7533	11.6317
1981	11.6469	11.7058	11.5881	11.7082	11.6797	11.6468	11.7388	11.8016	11.6760
1982	11.6779	11.7381	11.6177	11.7494	11.8127	11.6862	11.7851	11.8499	11.7203
1983	11.7089	11.7704	11.6473	11.7906	11.8556	11.7256	11.8313	11.8981	11.7645
1984	11.7398	11.8027	11.6770	11.8318	11.8986	11.7649	11.8776	11.9464	11.8087
1985	11.7708	11.8350	11.7066	11.8730	11.9416	11.8043	11.9239	11.9947	11.8530
1986	11.8017	11.8673	11.7362	11.9141	11.9846	11.8437	11.9701	12.0431	11.8972
1987	11.8327	11.8996	11.7658	11.9553	12.0276	11.8830	12.0164	12.0914	11.9414
1988	11.8637	11.9319	11.7954	11.9965	12.0706	11.9224	12.0627	12.1397	11.9856
1989	11.8946	11.9642	11.8250	12.0377	12.1136	11.9617	12.1089	12.1880	12.0298
1990	11.9256	11.9965	11.8546	12.0788	12.1566	12.0011	12.1552	12.2364	12.0740
1991	11.9565	12.0289	11.8842	12.1200	12.1996	12.0404	12.2015	12.2847	12.1182
1992	11.9875	12.0612	11.9138	12.1612	12.2426	12.0798	12.2477	12.3331	12.1624
1993	12.0185	12.0935	11.9434	12.2024	12.2857	12.1191	12.2940	12.3814	12.2066
1994	12.0494	12.1259	11.9730	12.2436	12.3287	12.1584	12.3403	12.4298	12.2508
1995	12.0804	12.1582	12.0025	12.2847	12.3717	12.1977	12.3865	12.4781	12.2950
1996	12.1113	12.1905	12.0321	12.3259	12.4148	12.2371	12.4328	12.5265	12.3391
1997	12.1423	12.2229	12.0617	12.3671	12.4578	12.2764	12.4791	12.5749	12.3833
1998	12.1732	12.2552	12.0913	12.4083	12.5008	12.3157	12.5253	12.6232	12.4275
1999	12.2042	12.2876	12.1208	12.4494	12.5439	12.3550	12.5716	12.6716	12.4716
2000	12.2352	12.3199	12.1504	12.4906	12.5869	12.3943	12.6179	12.7200	12.5158

TABLE 10

Forecasts of Peak Demand From Gross National Product (GNP) Projections.
(log linear form of equations)

Level of Confidence - 95%

All forecast values are log transformed values (to base e)

YEAR	5% Growth rate of GNP: Point forecasts	5% Growth rate of GNP: Upper limit of forecasts	5% Growth rate of GNP: Lower limit of forecasts	6% Growth rate of GNP: Point forecasts	6% Growth rate of GNP: Upper limit of forecasts	6% Growth rate of GNP: Lower limit of forecasts	7% Growth rate of GNP: Point forecasts	7% Growth rate of GNP: Upper limit of forecasts	7% Growth rate of GNP: Lower limit of forecasts
1976	9.5312	9.6752	9.3872	9.5416	9.6862	9.3971	9.5520	9.6971	9.4069
1977	9.5850	9.7318	9.4381	9.6059	9.7538	9.4579	9.6266	9.7756	9.4775
1978	9.6388	9.7885	9.4890	9.6701	9.8215	9.5187	9.7012	9.8542	9.5481
1979	9.6925	9.8451	9.5400	9.7343	9.8891	9.5795	9.7757	9.9328	9.6187
1980	9.7463	9.9901	9.5909	9.7986	9.9568	9.6403	9.8503	10.0114	9.6892
1981	9.8001	9.9585	9.6417	9.8628	10.0246	9.7010	9.9249	10.0901	9.7598
1982	9.8539	10.0152	9.6926	9.9270	10.0923	9.7618	9.9995	10.1687	9.8303
1983	9.9077	10.0719	9.7435	9.9913	10.1601	9.8225	10.0741	10.2474	9.9007
1984	9.9615	10.1286	9.7943	10.0555	10.2278	9.8832	10.1487	10.3262	9.9712
1985	10.0153	10.1854	9.8451	10.1197	10.2956	9.9439	10.2233	10.4049	10.0416
1986	10.0690	10.2421	9.8960	10.1840	10.3635	10.0045	10.2978	10.4837	10.1120
1987	10.1228	10.2989	9.9468	10.2482	10.4313	10.0652	10.3724	10.5625	10.1824
1988	10.1766	10.3557	9.9976	10.3125	10.4991	10.1258	10.4470	10.6413	10.2528
1989	10.2304	10.4125	10.0483	10.3767	10.5670	10.1864	10.5216	10.7201	10.3231
1990	10.2842	10.4693	10.0991	10.4409	10.6348	10.2470	10.5962	10.7985	10.3935
1991	10.3380	10.5261	10.1499	10.5052	10.7027	10.3076	10.6708	10.8777	10.4638
1992	10.3918	10.5829	10.2006	10.5694	10.7706	10.3682	10.7454	10.9566	10.5341
1993	10.4455	10.6397	10.2514	10.6336	10.8385	10.4288	10.8200	11.0355	10.6045
1994	10.4993	10.6965	10.3021	10.6979	10.9064	10.4894	10.8945	11.1143	10.6747
1995	10.5531	10.7534	10.3529	10.7621	10.9743	10.5499	10.9691	11.1932	10.7450
1996	10.6069	10.8102	10.4036	10.8263	11.0422	10.6105	11.0437	11.2721	10.8153
1997	10.6607	10.8671	10.4543	10.8906	11.1101	10.6710	11.1183	11.3510	10.8856
1998	10.7145	10.9239	10.5050	10.9548	11.1781	10.7315	11.1929	11.4299	10.9558
1999	10.7683	10.9808	10.5557	11.0190	11.2460	10.7921	11.2675	11.5089	11.0261
2000	10.8220	11.0377	10.6064	11.0833	11.3140	10.8526	11.3421	11.5878	11.0963

TABLE 11

Forecasts of Peak Demand from Industrial Output Projections
(Log linear form of equations)

Level of confidence 95%   All forecast values are log transformed values (to base e)

Annual growth rate of industrial output.

YEAR	8%			9%			10		
	Point-fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point. fore-casts	Upper limit of fore-casts	Lower limit of fore-casts	Point fore-casts	Upper limit of fore-casts	Lower limit of forecasts
1976	9.8651	9.9295	9.8007	9.8842	9.9490	9.8194	9.9032	9.9684	9.8380
1977	10.0251	10.0930	9.9572	10.0634	10.1321	9.9947	10.1014	10.1709	10.0319
1978	10.1851	10.2565	10.1138	10.2426	10.3153	10.1700	10.2996	10.3735	10.2257
1979	10.3452	10.4201	10.2702	10.4218	10.4984	10.3452	10.4978	10.5761	10.4195
1980	10.5052	10.5837	10.4267	10.6010	10.6817	10.5204	10.6960	10.7788	10.6132
1981	10.6652	10.7473	10.5831	10.7802	10.8649	10.6955	10.8942	10.9814	10.8069
1982	10.8253	10.9110	10.7395	10.9594	11.0482	10.8706	11.0923	11.1842	11.0005
1983	10.9853	11.0747	10.8959	11.1386	11.2315	11.0457	11.2905	11.3869	11.1942
1984	11.1453	11.2384	11.0523	11.3178	11.4148	11.2208	11.4887	11.5897	11.3878
1985	11.3054	11.4021	11.2086	11.4970	11.5982	11.3959	11.6869	11.7925	11.5814
1986	11.4254	11.5658	11.3650	11.6762	11.7815	11.5709	11.8871	11.9953	11.7749
1987	11.6254	11.7295	11.5213	11.8554	11.9649	11.7459	12.0833	12.1981	11.9685
1988	11.7855	11.8933	11.6776	12.0346	12.1483	11.9209	12.2815	12.4009	12.1620
1989	11.9455	12.0571	11.8339	12.2138	12.3317	12.0960	12.4797	12.6038	12.3556
1990	12.1055	12.2208	11.9902	12.3930	12.5151	12.2709	12.6779	12.8066	12.5491
1991	12.2656	12.3846	12.1465	12.5722	12.6958	12.4459	12.8760	13.0095	12.7426
1992	12.4256	12.5484	12.3028	12.7514	12.8819	12.6209	13.0742	13.2123	12.9361
1993	12.5856	12.7122	12.4590	12.9306	13.0653	12.7959	13.2724	13.4152	13.1296
1994	12.7457	12.8760	12.6153	13.1098	13.2488	12.9708	13.4706	13.6181	13.3231
1995	12.9057	13.0398	12.7715	13.2890	13.4322	13.1458	13.6688	13.8121	13.5166
1996	13.0657	13.2036	12.9278	13.4682	13.6156	13.3207	13.8670	14.0239	13.7100
1997	13.2258	13.3675	13.0840	13.6474	13.7991	13.4957	14.0652	14.2268	13.9035
1998	13.3858	13.5313	13.2403	13.8266	13.9826	13.6706	14.2634	14.4297	14.0970
1999	13.5458	13.6951	13.3965	14.0058	14.1660	13.8455	14.4615	14.6326	14.2904
2000	13.7059	13.8590	13.5528	14.1850	14.3495	14.0205	14.6597	14.8356	14.4839

TABLE 12

Forecasts of Peak Demand from Population Projections.
(Log linear form of equations)

Level of Confidence - 95%　　All forecast values are log transformed values(to base e)

YEAR	Annual growth rate of population: 1.5% Point forecasts	1.5% Upper limit of forecasts	1.5% Lower limit of forecasts	2.0% Point forecasts	2.0% Upper limit of forecasts	2.0% Lower limit of forecasts	2.25% Point forecasts	2.25% Upper limit of forecasts	2.25% Lower limit of forecasts
1976	9.7683	9.8500	9.6866	9.7919	9.8743	9.7096	9.8037	9.8864	9.7210
1977	9.9115	9.9236	9.7562	9.8871	9.9722	9.8021	9.9107	9.9964	9.8249
1978	9.9830	9.9972	9.8257	9.9823	10.0701	9.8946	10.0176	10.1064	9.9289
1979	10.0546	10.0708	9.8953	10.0775	10.1680	9.9871	10.1246	10.2165	10.0328
1980	10.1262	10.1444	9.9648	10.1727	10.2660	10.0795	10.2316	10.3265	10.1367
1981	10.1978	10.2181	10.0343	10.2679	10.3639	10.1720	10.3386	10.4366	10.2405
1982	10.2694	10.2917	10.1038	10.3631	10.4619	10.2644	10.4455	10.5467	10.3444
1983	10.3409	10.3654	10.1733	10.4584	10.5599	10.3568	10.5525	10.6568	10.4482
1984	10.4125	10.4390	10.2428	10.5536	10.6579	10.4492	10.6595	10.7670	10.5520
1985	10.4841	10.5127	10.3123	10.6488	10.7559	10.5416	10.7664	10.8771	10.6558
1986	10.5557	10.5864	10.3818	10.7440	10.8539	10.6340	10.8734	10.9873	10.7596
1987	10.6297	10.6601	10.4513	10.8392	10.9520	10.7263	10.9804	11.0974	10.8633
1988	10.6988	10.7338	10.5207	10.9344	11.0500	10.8187	11.0874	11.2076	10.9671
1989	10.7704	10.8075	10.5902	11.0296	11.1481	10.9110	11.1943	11.3178	11.0709
1990	10.8420	10.8812	10.6596	11.1248	11.2462	11.0034	11.3013	11.4280	11.1746
1991	10.9136	10.9549	10.7291	11.2200	11.3442	11.0957	11.4083	11.5382	11.2783
1992	10.9851	11.0286	10.7985	11.3152	11.4423	11.1880	11.5153	11.6484	11.3821
1993	11.0567	11.1023	10.8680	11.4104	11.5404	11.2804	11.6222	11.7587	11.4858
1994	11.1283	11.1761	10.9374	11.5056	11.6385	11.3727	11.7292	11.8689	11.5895
1995	11.1999	11.2498	11.0068	11.6008	11.7366	11.4650	11.8362	11.9791	11.6932
1996	11.2715	11.3235	11.0762	11.6960	11.8347	11.5573	11.9431	12.0894	11.7969
1997	11.3430	11.3973	11.1456	11.7912	11.9328	11.6496	12.0501	12.1996	11.9006
1998	11.3649	11.4710	11.2151	11.8864	12.0309	11.7419	12.1571	12.3099	12.0043
1999	11.4146	11.5447	11.2845	11.9816	12.1290	11.8342	12.2641	12.4202	12.1079
2000	11.4862	11.6185	11.3539	12.0768	12.2272	11.9264	12.3710	12.5304	12.2116

obtained from population projections are likely to be. less reliable. The forecast obtained from GNP and industrial output do not differ widely. However, the GNP based forecasts are lower in magnitude as compared to industrial production based forecasts. This can be attributed to the fact that GNP includes output from all sectors of the economy and all these sectors are not heavy consumers of electricity in comparison consumption in the industrial sector. A five percent growth in GNP does not correspond to eight percent growth in industrial output.

The 95% probability limit forecasts provide us quantitative information on risks associated with plans based on these forecasts. This is one of the chief advantages of adding probability dimensions to forecasting. The additional requirement of electricity energy over that of 1974 - 75.for the terminal year (1979 - 80) of fifth plan of India is expected to be approximately 25300 Million KWH and for the terminal year of 6th plan about 60225 million KWH. The additional peak power demand based on 5% rate of growth of GNP for terminal year of fifth plan and sixth plan are 4704, and 10195 M.W. respectively. This implies that these additional amounts of capacities have to be installed for meeting peak demands

6.3 RESULTS FOR ECONOMETRIC ANALYSIS OF DEMAND FOR ELECTRICITY IN THE INDUSTRIAL SECTOR

In this section the results of the econometric analysis of demand for electricity in the industrial sector have been presented. The analysis has been carried out for the following industries, namely Food, Textiles, Iron and Steel, Chemical, Non-ferrous metals, Engineering, Vehicles, Paper, Mining and Quarrying.

The ordinary least squares techniques has been used for estimating the econometric equations. The results of the analysis provide us quantitative estimates of elasticities and lag terms. Further, the results provide us with information for carrying out the following kinds of comparisons.

1. Direct comparison of goodness of fit of particular forms of equations for a particular industry group. To the extent that these equations can be considered as alternatives, we can draw conclusions on the relative merits of each.

2. Inter-industry comparison of size and significance of parameters and goodness of fit of equations. This type of analysis could be followed up by a cross section analysis by relating the inter-industry differences to economic factors. An examination can be made of as to how differences underlying economic

environment in which industries operate condition the responsiveness of their electricity demand to changes in certain causal variables over time.

Besides these two types of comparisons, the effects of inclusion of particular variables on the general fit of equations and on the size and significance of other parameters have been studied.

The results obtained are interpreted using the above stated basis of comparisons. The interpretation of the results are presented in a later section.

6.3.1 Evaluation Of Goodness Of Fit Of Equations

The procedure that has been followed in comparing the goodness of fit of equations is to calculate for each equation (over all nine industry groups) the mean R^2 (Multiple Correlation Coefficient) and the mean percentage of standard error (S.E) of equations togather with the corresponding standard deviation of these means. The equations are ranked on the basis of (a) highest R^2, (b) lowest standard error of equation (c) lowest standard deviation of R^2, (d) lowest standard deviation of the standard error of equation.

The degrees of freedom vary between equations. Hence it is possible for R^2 and S.E of equationsto give

conflicting results. The extent of ambiguity can be estimated by determining the rank correlation coefficient between ranking on the basis of (a) and (b). It is found that this coefficient is 0.925 and therefore there is little significant ambiguity. Comparison of R^2 with its S.D.; and S.E. of equation with its S.D., indicates the extent to which equations perform consistently well or poorly over all industries. In fact it is found that those equations which perform best perform consistently best having the smallest standard deviations. The rank correlation coefficient between R^2 and its S.D.; S.E. of equation and its S.D. will give an idea between the relation between relative consistency and relative goodness of fit. Again we find reasonably high correlations. Let ρ denote the rank correlation coefficient. The following results have been obtained for ρ .

$$\rho\,(R^2,\ \text{S.E of equation}) = 0.925$$

$$\rho\,(R^2,\ \text{S.D. of } (R^2)) = 0.946$$

$$\rho\,(\text{S.E.};\ \text{S.D. of (S.E)}) = 0.729$$

An examination of Table 13 indicates that the equation ranked highest on R^2 is also ranked highest on S.E. of equation and it also tends to have lowest S.D. implying greater consistency over all industry groups. Although there is some switching in position as the

TABLE 13

Results of goodness of fit of equations for industrial electricity demand.

No. of the Equation	Mean R^2	Rank No.	Mean S.E of Equation(%)	Rank No.	S.D. of R^2	Rank No.	S.D. of S.E.	Rank No.
4.12	0.981	1	5.12	2	0.054	2	2.01	2
4.13	0.979	2	4.53	1	0.048	1	1.79	1
4.16	0.975	3	5.79	4	0.068	3	2.46	5
4.11	0.974	4	5.46	3	0.079	4	2.28	4
4.18	0.970	5	6.07	5	0.083	5	2.67	7
4.15	0.968	6	6.24	6	0.094	6	3.16	10
4.17	0.963	7	6.43	7	0.099	7	2.58	6
4.19	0.953	8	6.96	9	0.083	5	2.13	3
4.25	0.949	9	7.34	11	0.113	8	2.84	8
4.23	0.944	10	7.01	10	0.120	9	2.99	9
4.24	0.938	11	7.34	11	0.134	10	2.84	8
4.26	0.902	12	6.78	8	0.148	11	2.46	5

Explanation of symbols:

R^2 - Coefficient of determination of equation.

S.E - Standard error.

S.D. - Standard deviation.

criteria are changed, yet results between groups of industries are more stable. Equations(4.12 & 4.13) are marginally better than equations(4.15)to(4.23)on all the four criteria. However, the differences are very small. In view of the fact that the record group is subject to measurement error, the results of the record group are good. The inclusion of a time variable in equation(4.16)may possibly account for goodness of fit there, while in the rest of this group the presence of the coal variable has a considerable effect on goodness of fit. The third group of equations corresponding to Model 3 are clearly consistently inferior on the criteria selected.

However the narrowness of this basis of comparison and smallness of differences, especially between group of equations corresponding to Model 1 and Model 2 do not permit a firm conclusion whether one set of equations (and therefore the hypothesis involved in that model) is superior to another, for purposes of forecasting and analysis. Similarly within the groups of equations differences are even smaller and comparison even less conclusive.

6.3.2 Explanatory Power Of Variables

The explanatory power of an equation included is determined by explanatory power of the variables and the form of relationship that it is assumed to take.

Though R^2 is an important criterion for testing the goodness of fit and in assesing the relative usefulness of any set of equations, in this analysis this should not be the only criterion which should be used. Because of inclusion of a lagged dependent variable it is possible for an equation to have high R^2 but with none of the meaningful explanatory variables significant. This may be either due to lack of explanatory power of variables concerned or due to presence of multicollinearity, which is then a wrongly specified equation. Hence we adopt two further bases of comparison. These correspond to (1) the general level of significance of variables in an equation, and (2) the degree to which multicollinearity appears to be present.

Clearly it would require a large amount of space to present the whole set of parameter estimates for each variable in each equation in each industry group. Therefore, we present a summarised picture of it in Table 14. This table gives for each variable in each equation the number of industry groups in which the parameter estimated was significant at the 0.95 level. From the table we get an idea of the relative explanatory power of particular variable. From Table 14 it is clear that apart from lagged dependent variable, the variables which are the most common determinants of changes in electricity

TABLE 14

Comparison of explanatory power of variables for industrial electricity demand

Variable equation number	Q_t	Log Q_t	Log $(P/F)_t$	Log $(P/W)_t$	t	C_t	C_{t-1}	$(M/Q)_t$	Log $(M/Q)_t$	$(I/Q)_t$	Log $(I/Q)_t$	Log D_{t-1}	Z_{t-1}	Log $(D/Q)_{t-1}$
4.11		6										9		
4.12		8	5									9		
4.13		4		5								[illegible]		
4.15	6					4	2						8	
4.16	6				3	6	0						5	
4.17	7					7							8	
4.18	5					5		4					8	
4.19						5				3			9	
4.23									1					9
4.24											5			9
4.25		4												9
4.26			3											9

Explanation of Symbols

Q_t - Output of the industry in period t

W - Average wage rates of workers in the industry

M - Number of employees in the industry

C_t - Coal consumed in period t

Z_t - Consumption of electricity in coal equivalent tons

P - Price of electricity

F - Full price index

I - Gross fixed capital stock formation

D_t - Consumption of electricity in period t

t - Time period

6.3.3 Inter-Industry Elasticity Coefficients Comparison

For inter-industry comparisons, we select from the large number of varying estimates of coefficients of a particular variable, the one which is considered best for a particular industry group on the criterion given above.

Table 15 gives some selected results from the first two groups of equations. It can be seen from table 9 that the industries fall broadly into two categories. Those industries for which significant index of production elasticities are greater than unity. The industries which belong to this category are Food, Nonferrous metals, Textiles, Iron and Steel, Vehicles and Mining and Quarrying.

None of these industries have significant fuel price elasticities. The price wage relative is important in three classes namely Textiles, Nonferrous metals, Iron and Steel. The remaining industries do not show any responsiveness of electricity demand to changes in price relative to wage rates.

It is noticeable that food, and nonferrous metal industries have non-significant coal variables. This suggests absence of developments affecting the pattern of fuel use such as have occured in other industries. In

TABLE 15

Estimated elasticity coefficients of industrial electricity demand.

INDUSTRY GROUP	+ $E_{D,Q}$	+ $E_{D,(P/F)}$	++ $E_{D,(P/W)}$	** $E_{Z,(I/Q)}$	+++ $E_{Z,(I/Q)}$	*** $E_{Z,C}$	**** $E_{Z,t}$	+ λ	Mean S.E.(%)
Food	2.571	-0.415*	-1.046*	2.493	-2.541	-0.056*	0.348	0.632	3.9
Chemicals	0.821	-1.069*	-1.096*	-1.607*	-8.312	0.237*	0.291	0.148	2.8
Non ferrous metals	1.310	-0.843*	-2.543	-0.103*	2.367*	0.031*	0.173	0.246	4.2
Iron & Steel	1.507	-2.257*	-2.722	-0.378	-3.581*	-0.397	0.146*	0.115	3.4
Engineering	0.944	-0.588*	-0.712	-0.473*	2.206	-0.156	0.002*	0.490	3.9
Vehicle	1.216	-1.428*	-1.280*	-0.144*	-0.127*	-1.285	0.216*	0.200	4.8
Textiles	1.307	-1.651*	-1.432	0.570*	-2.536*	-0.403	0.090*	0.148	5.8
Paper	0.746	-1.083*	-0.793*	-0.854	2.732*	-0.275	0.050	0.423	7.3
Mining & Quarrying	1.954	-2.017*	-1.307*	-0.025*	0.052*	0.383*	-0.025	-0.122	4.7

Explanation of Symbols

+ - Coefficient estimates obtained from Eq. 4.12
++ - " " " " Eq. 4.13
+++ - " " " " Eq. 4.18
** - " " " " Eq. 4.19
*** - " " " " Eq. 4.17
**** - " " " " Eq. 4.16

$E_{X,Y}$ - Elasticity of variable X with respect to variable Y

For Example: $E_{D,Q}$ - Elasticity of electricity demand D with respect to Q (similar interpration for other elasticities)

* - Denotes that these coefficients are not significant at 95% confidence level

S.E - Standard error

The Symbols D, P, F, t, I, Q, W, C, Z have been explained in Table No. 14

λ - The distributed lag parameter

consumption are the index of production, Q_t, and Coal and Coke consumption. The time variable in equation (4.16) is significant in surprisingly few cases and this may be due to intercorrelation with Q_t and C_t, both of which are strongly time trended . Apart from perhaps the (P/F) and (I/Q) variable in the Model 3 equations, there are no other variables with fairly general explanatory power. The following conclusions can be drawn from the result presented in the table.

(a) As between groups of equation there is little to choose between first group (Model 1) and second group (Model 2). Each group contains one very good equation, the remainder being fairly good or indifferent. It therefore appears that use of linear form and measurement of electricity in coal equivalent tons (C.E.T) makes less difference than what we had opined before. Both these groups are markedly superior to the third, partly because this third group relies on generally less successful variables and also because of the form assumed.

(b) In comparing equation between groups there is in each group one equation which is markedly better than others. In first group this is equation (4.12). In second group equation(4.17) has the highest general level of significance.

the absence of any other significant determinants it would seem that in these two industries capital stock effect of growth in industrial output accounts for the higher magnitude of electricity consumption rather than substitution.

Those industries which have significant index of production elasticities less than unity. The industries which belong to this group are Chemicals, Engineering and Paper.

The fuel price relative is not significant in case of Chemical, Paper and Engineering industries. For chemicals alone there is a high and significant elasticity of the (M/Q) variable suggesting a labour substitution effect, which in the presence of a non-significant (P/W) variable is probably the result of technological development. For Engineering and Paper industries the significance of coal elasticity is difficult to interpret. This may be in part due to substitution as a result of relative price movements. For these industries, the results do not show any discrimination between price and technological substitution effects.

Finally we consider the results of the consumption ratio form in Table 16. The general lack of significance of variables in this case reflects the inappropriateness of the assumption that elasticity of electricity demand with respect to output is unity. On the other hand,

TABLE 16

Results of elasticity coefficients obtained from Model - 3 Equations

Industry Group	$E_{D/Q,P/F}$	$E_{D/Q,P/W}$	$E_{D/Q,M/Q}$	$E_{D/Q,I/Q}$
Food	-1.917	-1.456	-2.206	1.094
Chemicals	-0.587	-0.113*	-0.071*	0.014*
Non ferrous Metals	-0.887*	-1.044*	0.298*	-0.062*
Iron & Steel	-1.541*	-0.690*	0.179*	0.616*
Engineering	-0.416	-0.319	-0.149*	-0.181*
Vehicles	-1.498	-1.389	0.468*	0.008*
Textiles	-1.425	-1.090*	0.462*	0.195*
Paper	-0.401*	-0.029*	0.227*	0.126*
Mining & Quarrying	-1.059*	-0.518*	-0.612*	1.035*

* Indicates that these coefficients are insignificant at 0.95 level of significance.

$E_{D/Q,P/F}$ - Elasticity of D/Q with respect to P/F.

Similar interpretations are to be made for other elasticities.

where in Table 15 the index of production elasticity is relatively close to unity, the results on significance of variables in the first and third models, tend to coincide particularly for relative price variables. Nevertheless the poor results of the model 3 which is based on consumption ratio suggest that the first two models constitute a better approach to electricity demand analysis.

6.3.4 Interpretation Of Results

The main points that we will discuss are the inter-industry difference in results and the results for the relative price variables. However, before discussing the results we consider the various statistical reasons for cautious interpretation of the results.

The use of distributed lags may give rise to autocorrelation with consequent bias in estimates of standard errors and therefore it may result in mistaken conclusion on the significance of parameter estimates. The D-W (Durbin - Watson) statistic has indicated that autocorrelation is present.

Further although some attempt was made in the formulation of equations to minimise the risk of multicollinearity between independent variables, it is evident from the simple correlation matrix of all variables that

TABLE 17

The effect of correlation between variables on levels of significance

Industry Group	Simple correlation between Q_t (Index of production) and P/	P/W	't' value of coefficient of Q_t in equation $D_t = f(Q_t, P/F)$	$D_t = f(Q_t, P/W)$	Differences (2)-(1)	(4)-(3)
	(1)	(2)	(3)	(4)	(5)	(6)
Iron & Steel	-0.546	-0.769	7.65	2.9	-0.223	4.75
Mining & Quarrying	0.678	0.753	6.97	1.94	0.075	5.03
Chemicals	0.858	-0.965	4.93	1.05	-1.823	3.88
Non ferrous metals	-0.795	-0.897	8.19	2.07	-0.102	6.12
Paper	-0.820	-0.906	3.97	1.758	-0.086	2.20

The symbols P/F and P/W have been explained in Table No. 14

The level of significance of variables and labour intensity of industries are tabulated in Table - 18.

There are clear differences between labour-intensive and capital intensive industries in respect of the significance of output and coal-consumption variables. The output variable is significant in all the industries concerned, where as the coal variable is not significant in all industries. This mean that the relationship between output and electricity demand may to a large extent be determined by the capital-stock characteristics. The explanation of the pattern of significance of coal consumption variable lies in the technologies of the industries concerned.

The results indicate that ($\frac{P}{W}$) variable is not significant in mining and quarying, vehicles, paper, food, and chemicals. This suggests that the possibility of substitution of electricity for labour in these industries are necessarily circumscribed by the fact that electricity may often be embodied in expensive machinery. Although relative changes in electricity price and wages may to some extent lead to mechanisation, it still may represent a proportionately small change in the relative costs of production. However some industries like textile, engineering, non-ferrous metals, and iron and steel do appear to have some price-wage responsiveness to electricity

TABLE 18

Levels of significance and labour intensity

Industry Group	Variables					
	Q	P/F	P/W	I/Q	M/Q	C
Mining and Quarrying	+	-	-	-	-	+
Vehicles	+	-	-	-	-	-
Textiles	+	-	+	-	-	+
Engineering	+	-	+	-	+	-
Paper	+	-	-	+	-	+
Non ferrous metals	+	-	+	-	-	-
Iron & Steel	+	-	-	+	+	-
Food	+	-	-	+	+	-
Chemicals	+	-	-	-	+	-

\+ Denotes that these variables are significant.

\- Denotes that these variables are non-significant.

Note: The industries are ranked in order of increasing capital intensity.

Symbols Q, P, F, W, I, M, C have been explained in Table 14.

demand. These conclusions are of course subject to reservations stemming from the results tabulated in Table 17.

The fuel price relative is almost insignificant in all the industry groups. This is expected. From the available data on the value of electricity costs per thousand rupees of output, there does not appear to be any importance of electricity as a cost factor and hence there is no responsiveness of demand to changes in price. For industries where (P/F) variable is not significant there are two other possible explanations. The first is that there is a strong complimentarity between fuels and capital equipment. It is not possible to substitute among fuels without changing type of plant and machinery used, so that other things being equal changes in relative fuel costs has little effect on relative costs of producing from different equipment. These relative costs can change in a way that it may not take advantage of favourable relative price movements. The second explanation is a statistical one. The fuel price relative is derived from general price index for fuels used by manufacturing industries and so is same for all industries. Although this index is a good indicator of the general level of price paid, the actual price paid by firms vary according to area and size of firms and also between industries. There is a degree of

approximation involved which may imply that results do not measure actual response to price changes. This second explanation raises the question of the reliability of significant estimates obtained. Can we infact infer a causal relationship or do the significant elasticities simply represent correlated time trends of two variables. To test this hypothesis equation (4.14) was estimated with (P/F) replaced by time trend t. The results for this equation for all industries showed that R^2, and index of production elasticities were roughly same as in equation (4.13). The simple correlation between price and time was 0.941. Thus it is difficult to refute that the price variable has acted as a time trend.

Finally as noted above there appear to exist a relation between capital intensity and significance of index of production. It is also noticable that capital intensive industries have non significant price effects, and have index of production elasticity greater than unity. This tends to confirm fairly closely to our hypothesis suggested in section 4.2.1 of Chapter IV. However, there is a qualification which must be made on statistical grounds. The output for each group of industry is a base weighted index for broad group of products, while electricity consumption is simply unweighted total. Clearly therefore it may be possible for equal proportionate

growth in electricity consumption and output to take place in each sub-industry while the system of weighing for index of production could lead to a different relationship in aggregate.

CONCLUSIONS

The main conclusion of this analysis is that the relative price changes are not important determinants of growth in industrial electricity consumption. The chief determinants are growth in industrial output and changes in technology taken at a face value, the results for the relative price variable suggest that price elasticity of demand is highly insignificant. This, if valid, would seem to have a relevance for current developments in the energy economy. It suggests that considerable changes in relative price may have to be necessary to offset even partially the growth of industrial electricity demand and that other form of energy are highly unlikely to have any significant effect on electricity's share in the industrial energy consumption.

On the other hand the results may have led us to an incorrect interpretation. The possibility that the price variable mostly acted as a time trend has been demonstrated so that where the variable is significant we cannot necessarily infer that there was a significant price effect. At the same time we also cannot infer that

where the variable is not significant, there would not in fact be a significant price elasticity. This arises because the price variable has been derived from wholesale price index of fuels used by manufacturing industries. It does not necessarily reflect the actual price paid by a particular set of industrial consumers, of a given size and region. Thus the results for this variable are no more than first approximations. Given that on technological grounds there is no reason to expect the absence of an important overlap of areas of substitution between electricity and other fuels, the statistical limitations imposed on the analysis go some way in explaining what must be to the theoretical economist the rather surprisingly unimportance of price.

However, the discussions and results have also brought out the important part that must be played by the associated equipment in any business decision on the choice of fuels. Unlike raw material inputs fuels are demanded for services they perform within the production activity. Electricity is a part of a group of complimentary goods composed of the various types of electricity consuming capital. The indices by which the relative price have been measured, because they are based on electricity and other fuels alone may not be an adequate indicator of price relatives of electricity's whole group of compliments. It is the price elasticity of these compliments which is more relevant to theoretical expectation of rational behaviour.

6.4 RESULTS FOR STOCHASTIC TIME SERIES ANALYSIS AND FORECASTING OF ELECTRICITY DEMAND

In this section we present the results for stochastic time series analysis of electricity demand and prediction of the future values of the time series. Results are given in the order of the procedure adopted for identifying, estimating, validating the model. The future values of electricity demand are forecasted by using the selected model.

Before presenting the results we discuss about the data used for the study.

The data on time series of peak power demand and energy demand have been collected from various sources. The sources of data are presented in Appendix A. Both the time-series data of peak demand and energy demand are discrete in nature. We denote the time-series of electricity demand as series A and peak power demand as series B. Series A is a time series of fortyseven annual observations of electricity consumption (in million KWH units) and series B is a series of fortyseven observations of annual peak power demand (in MW) for the whole of India.

6.4.1 Identification Of Trend

A graphical plot of series A and B indicates the existence of a distinct trend. The trend appears to be an exponential function of time.

The trend components of series A and B are assumed to be of the following forms:

Model I (Polynomial trend component)

$$E_t^T = \alpha_0 \; e^{\alpha_1 t} \; e^{\alpha_2 t^2} \qquad (6.1)$$

$$P\,D_t^T = \beta_0 \; e^{\beta_1 t} \; e^{\beta_2 t^2}$$

Model II (Linear trend component)

The linear trend for series A and B are assumed to be of the following forms.

$$E_t^T = a_0 \, e^{a_1 t}, \qquad (6.2)$$

$$P\,D_t^T = b_0 \, e^{b_1 t}$$

where

E_t^T = Trend component of series A at time t

$P\,D_t^T$ = Trend component of series B at time t

a, b, α, β etc. are the parameters

Taking logarithm of both sides of equations (6.1) and (6.2) we have

$$
\begin{aligned}
\text{Log } E_t^{\,T} &= \log \alpha_0 + \alpha_1 t + \alpha_2 t^2 \\
\text{Log } P\, D_t^{T} &= \log \beta_0 + \beta_1 t + \beta_2 t^2 \qquad (6.3) \\
\text{Log } E_t^{\,T} &= \text{Log } a_0 + a_1 t \\
\text{Log } P\, D_t^{\,T} &= \text{Log } b_0 + b_1 t
\end{aligned}
$$

The trend component is a monotonically increasing function of time, without any fluctuation around a mean trend line. This leads us to tentatively suggest that cyclic components may be absent. The presence of cyclic components have to be tested by correlation and spectral analysis.

The trend components are estimated by multiple regression analysis. The ordinary least squares technique have been used forthis purpose. The estimated equations are presented below.

Trend models estimated for series A. (Energy Demand)

Model 1A (Polynomial Trend)

$$E_t^{\,T} = \underset{(173.82)}{2.977} + \underset{(18.743)}{0.030\, t} + \underset{(9.033)}{0.0003}\; t^2 \qquad \bar{R}^2 = 0.9964 \quad DW=1.734$$

Model 2 A (Linear Trend)

$$E_t^{\,T} = \underset{(154.021)}{2.8591} + \underset{(67.217)}{0.0452}\; t \qquad \bar{R}^2 = 0.9899 \quad DW=1.963$$

Trend models estimated for series B (peak demand)

Model 1B (Polynomial Trend)

$$P_t^T = \underset{(101.78)}{2.519} + \underset{(6.850)}{0.016}\, t + \underset{(9.555)}{0.0004}\, t^2 \qquad \bar{R}^2 = 0.9897 \quad DW = 1.766$$

Model 2B (Linear Trend)

$$P\,D_t^T = \underset{(84.002)}{2.339} + \underset{(37.947)}{0.038}\, t \qquad \bar{R}^2 = 0.9690 \quad DW = 1.598$$

where

$\bar{R}^2$ = Coefficient of determination of the equation.

DW = Durbin - Watson statistic.

The values in the parenthesis below the coefficients are their corresponding 't' values.

For Models 1A, 2A, 1B, 2B we observe that $\bar{R}^2$ values are quite high and the 't' values of the coefficients are significant. This leads us to suggest that the trend component is significant for both series A and B.

We have chosen the linear-trend for both series A and B, since the difference between $\bar{R}^2$ values for polynomial and linear-trend are negligible.

The linear trend components are then subtracted from the original time-series to obtain a trend-free series.

6.4.2 Results Of Identification Of Degree Of Differencing, Persistence And Cyclicity

Assuming that the resultant and trend-free series is an ARIMA process of order (p, d, q) the next step is to identify the values of p, d, q.

The principal tools that have been used for identifying (p, d, q) are the autocorrelation function partial autocorrelation function and power spectrum of the time series.

The autocorrelation and partial autocorrelation functions have been calculated for various degrees of differencing. The computational algorithm described in Box and Jenkins (45), has been used for this purpose.

To identify the degree of differencing (d) to produce a stationary ARMA process use is made of the properties of ACF and PACF of stationary series (45). The ACF for the non-differenced (d = 0) series A and B does not die out quickly indicating a non-stationary process. Hence these series are differenced once and ACF and PACF for different lag values are computed. The first ten autocorrelations and partial autocorrelations for the undifferenced and first differenced series are presented in Table 19. Tests of significance (45) for the ACF and PACF (whether they are effectively different from zero beyond a certain lag) have been conducted.

To identify the existence of periodic components, if any the smoothed spectra for degree of differencing zero, and one, for various lag values have been obtained for Series A and B by the method described by Jenkins and Watts (209). The computed raw and smooth spectra for various lag values are presented in Table 19.

From the characteristics of spectra that has been tabulated and recalling the properties of periodic components (209), we are led to suggest that no periodic components are present in both series A and B.

Based on the information provided by the ACF and PACF and spectra of series A and B for degree of differencing zero and one, and recalling the characteristics of ACF and PACF of various ARMA processes (45, 209, 217) the following orders of the ARIMA process have been identified. Since identification methods are rough procedures applied to a set of data to indicate the kind of representational model worthy of further investigation ten models have been tentatively selected for series A, and ten models have been chosen for series B. They are presented below.

ARIMA (p, d, q) models tentatively selected for Series A and B

1. (1, 0, 0)	2. (1, 0, 1)	3. (2, 0, 0)	4. (2, 0, 1)
5. (2, 0, 2)	6. (1, 0, 1)	7. (2, 1, 0)	8. (1, 1, 1)
9. (2, 1, 1)	10. (2, 1, 2).		

Results of initial estimation of parameters

The initial estimates of the parameters of the processes are required to be used as starting points for more refined iterative estimation procedures. The initial estimates are obtained from the estimates of ACF and PACF at the identification stage. The computational algorithm provided in Box and Jenkins (45) have been used to obtain the initial estimates. A summary of the initial estimates of parameters for the tentatively selected models are presented in Table 20.

Results of maximum likelihood estimation of parameters

The identification process having led to a tentative formulation of various models, the efficient maximum likelihood estimates of parameters are obtained by using the computational algorithm provided in (45). Marquardt's (230) non-linear estimation routine has been used for this purpose. The parameters and their variances estimated by the maximum likelihood principle for all the tentatively selected models are presented in Table 20.

Results of validation of models

The models are validated by means of diagnostic checks applied to the residuals of model. For details of the procedure reference is made to Box and Jenkins (45).

We only present the results of diagnostic checks for the model which is finally selected on the basis of these tests. The models that are finally selected for series A and B are identical ARIMA process of order (2, 1,0) with different values of parameters. The results of diagnostic checks are presented in Table 21.

From the auto correlation function of residuals it is observed that their values are statistically not different from zero. For a confidence level of 95% all the smooth spectra values lie between the upper and lower limits of and respectively, leading us to suggest that at the specified level of confidence the residuals are pure random. Further the chi-square test show that the calculated value of is less than the critical value at the specified level of significance.

Based on the results of the above tests for validation of model, we have selected the ARIMA models of order (2, 1, 0) for both series A and B.

The models that have been finally selected for representing the time series of electricity demand are presented below.

For Series A (Energy Demand)

$$(1 - 0.3316\, B - 0.2461\, B^2)\ (1 - B)\ Z_t = a_t$$

or

$$Z_t = 1.3316\ Z_{t-1} - 0.0855\ Z_{t-2} - 0.2461\ Z_{t-3} + a_t$$

$$\hat{\sigma}_a^2 = 0.883 \times 10^{-4}$$

For Series B (Peak Demand)

$$(1 - 0.3617\ B - 0.3631\ B^2)\ (1 - B)\ Z_t = a_t$$

or

$$Z_t = 1.3617\ Z_{t-1} + 0.0014\ Z_{t-2} - 0.3631\ Z_{t-3} + a_t$$

$$\hat{\sigma}_a^2 = 0.1847 \times 10^{-3}$$

The forecast values of peak demand and energy are obtained by using the demand models selected. Forecasts of annual peak and energy demand for India for the period 1976-2000 are presented in Table 22.

TABLE 19

Autocorrelation Function of Series A

Degrees of differencing	Lag Values 1	2	3	4	5	6	7	8	9	10
0	0.8736	0.7307	0.5686	0.4127	0.2206	0.0571	-0.0802	-0.1780	-0.2577	-0.3189
1	0.4116	0.3571	0.2241	0.3505	0.0464	0.0403	-0.0650	-0.0640	-0.0351	-0.0133
Autocorrelation Function of Series B										
0	0.8622	0.7101	0.5447	0.3717	0.1754	-0.0068	-0.1704	-0.3035	-0.4039	-0.4643
1	0.4622	0.4330	0.3864	0.3607	0.2206	-0.0705	-0.0560	-0.1071	-0.2455	-0.1702
Partial Autocorrelation Function of Series A										
0	0.8736	-0.1372	0.1610	-0.0689	-0.2691	-0.0183	-0.0248	-0.0004	-0.0408	-0.0958
1	0.4116	0.2259	0.0219	0.2450	-0.2429	-0.0588	-0.0751	-0.1060	0.1689	0.0207
Partial Autocorrelction Function of Series B										
0	0.8622	-0.1302	-0.1389	-0.1339	-0.2190	-0.1045	-0.1015	-0.0671	-0.0536	-0.0328
1	0.4622	0.2790	0.1560	0.1107	-0.0860	-0.2035	-0.2270	-0.1244	-0.1804	0.1226
Smooth Spectra of Series A										
0	0.6204	0.2154	0.0267	0.0178	0.0172	0.0099	0.0058	0.0041	0.0079	0.0099
1	0.0422	0.0184	0.0067	0.0096	0.0111	0.0085	0.0042	0.0035	0.0016	0.0142
Smooth Spectra of Series B										
0	1.3096	0.4752	0.0578	0.0450	0.0362	0.0246	0.0177	0.0134	0.0167	0.0176
1	0.1116	0.0338	0.0091	0.0186	0.0212	0.0177	0.0156	0.0170	0.0173	0.0203

TABLE 20

Initial and Maximum Likelihood Estimates of Parameters For Series A

	Initial Estimates		Maximum Likelihood Estimates					
Order of Process (p,d,q)	Parameters (ϕ, θ)	Residual variance a	Parameters (ϕ, θ)	U.S.S. a_t^2	S.E. of parameters	Residual mean	Residual variance	Mean spectra of residuals
1	2	3	4	5	6	7	8	9
(1, 0, 0)	0.8736	0.311×10^{-3}	0.9043	0.791×10^{-2}	0.0298	0.220×10^{-3}	0.113×10^{-3}	0.158×10^{-3}
(1, 0, 1)	0.8364	0.305×10^{-3}	0.8786	0.661×10^{-2}	0.0378	0.355×10^{-3}	0.975×10^{-4}	0.137×10^{-3}
	-0.1561	-	-0.3444	-	0.1493	-	-	-
(2, 0, 0)	0.9934	0.305×10^{-3}	1.5017	0.547×10^{-2}	0.1219	0.435×10^{-4}	0.885×10^{-4}	0.117×10^{-3}
	-0.1372	-	-0.5787	-	0.1098	-	-	-
(2, 0, 1)	2.1467	0.397×10^{-3}	1.9492	0.478×10^{-2}	0.0498	0.527×10^{-3}	0.108×10^{-3}	0.946×10^{-4}
	-1.1446	-	-0.9762	-	0.0527	-	-	-
	0.8312		0.7324	-	0.1275	-	-	-
(2, 0, 2)	1.4775	0.295×10^{-3}	1.8786	0.470×10^{-2}	0.0962	0.860×10^{-4}	0.109×10^{-3}	0.958×10^{-4}
	-0.5849	-	-0.9055	-	0.1003	-	-	-
	0.5094	-	0.6342	-	0.1950	-	-	-
	-0.1071	-	-0.0042	-	0.1668	-	-	-
(1, 1, 0)	0.4117	0.115×10^{-3}	0.4302	0.527×10^{-2}	0.1348	0.176×10^{-3}	0.938×10^{-4}	0.123×10^{-3}
(2, 1, 0)	0.3186	0.109×10^{-3}	0.3316	0.497×10^{-2}	0.1457	0.294×10^{-3}	0.883×10^{-4}	0.109×10^{-3}
	0.2260	-	0.2461	-	0.1469	-	-	-
(1, 1, 1)	0.8676	0.107×10^{-3}	0.8228	0.494×10^{-2}	0.1495	0.400×10^{-3}	0.112×10^{-3}	0.110×10^{-3}
	0.5581	-	0.4869	-	0.2320	-	-	-

Table continued on next page

TABLE 20 (continued)

1	2	3	4	5	6	7	8	9
(2, 1, 1)	0.4108	0.109×10^{-3}	0.6883	0.492×10^{-2}	0.4803	0.377×10^{-3}	0.114×10^{-3}	0.109×10^{-3}
	0.1880	-	0.0931	-	0.2868	-	-	-
	0.0963	-	0.3794	-	0.4710	-	-	-
(2, 1, 2)	-1.8203	0.747×10^{-3}	-0.0935	0.479×10^{-2}	0.0169	0.320×10^{-3}	0.114×10^{-3}	0.103×10^{-3}
	2.1239	-	0.6799	-	0.0346	-	-	-
	0.0879	-	-0.4060	-	0.0281	-	-	-
	0.1809	-	0.2769	-	0.1053	-	-	-

Initial and Maximum Likelihood Estimates of Parameters For Series B

1	2	3	4	5	6	7	8	9
(1, 0, 0)	0.8622	0.704×10^{-4}	0.9781	0.162×10^{-1}	0.0440	0.242×10^{-2}	0.246×10^{-3}	0.380×10^{-3}
(1, 0, 1)	0.8235	0.692×10^{-3}	0.9431	0.138×10^{-1}	0.0542	0.171×10^{-2}	0.307×10^{-3}	0.318×10^{-3}
	-0.1502	-	-0.3493	-	0.1499	-	-	-
(2, 0, 0)	0.9746	0.692×10^{-3}	1.5207	0.118×10^{-1}	0.1329	0.136×10^{-2}	0.262×10^{-3}	0.261×10^{-3}
	-0.1302	-	-0.5858	-	0.1323	-	-	-
(2, 0, 1)	*	0.948×10^{-3}	*	*	*	*	*	*
	-1.0344	-	-	-	-	-	-	-
(2, 0, 2)	*	0.663×10^{-3}	*	*	*	*	*	*
(1, 1, 0)	0.4622	0.265×10^{-3}	0.5123	0.118×10^{-1}	0.1033	0.162×10^{-3}	0.263×10^{-3}	0.271×10^{-3}
(2, 1, 0)	0.3332	0.244×10^{-3}	0.3617	0.104×10^{-1}	0.1426	0.390×10^{-3}	0.184×10^{-3}	0.225×10^{-3}
	0.2790	-	0.3631	-	0.1463	-	-	-

(continued on next page)

TABLE 20 (continued)

1	2	3	4	5	6	7	8	9
(1, 1, 1)	0.9369	0.232×10^{-3}	0.9243	0.101×10^{-1}	0.0868	0.797×10^{-3}	0.230×10^{-3}	0.225×10^{-3}
	0.6234	-	0.5436	-	0.1764	-	-	-
(2, 1, 1)	0.8488	0.236×10^{-3}	0.6947	0.986×10^{-2}	0.2964	0.865×10^{-3}	0.229×10^{-3}	0.215×10^{-3}
	0.0407	-	0.2153	-	0.2367	-	-	-
	0.5479	-	0.4215	-	0.2868	-	-	-
(2, 1, 2)	0.0695	0.235×10^{-3}	-0.0456	0.981×10^{-2}	0.0961	0.973×10^{-3}	0.233×10^{-3}	0.208×10^{-3}
	0.7708	-	0.9256	-	0.1116	-	-	-
	-0.1456	-	-0.4048	-	0.1667	-	-	-
	0.4983	-	0.5170	-	0.1962	-	-	-

Explanation of Symbols:

ϕ - Parameters of AR process.

θ - Parameters of MA process.

a_t^2 = Unconditional sum of squares

* - Indicates the iteration in initial estimate programme does not converge.

S.E - Standard Error

TABLE 21

Autocorrelation Function and Smooth Spectrum of Residuals of Models Finally Selected.

Lag	SERIES A ACF	SERIES A Smooth Spectra	SERIES B ACF	SERIES B Smooth Spectra
1	-0.0186	0.1065×10^{-3}	-0.1080	0.1862×10^{-3}
2	-0.0834	0.1079×10^{-3}	-0.1759	0.0604×10^{-3}
3	-0.0183	0.0791×10^{-3}	0.1610	0.0989×10^{-3}
4	0.3398	0.0707×10^{-3}	0.2341	0.2843×10^{-3}
5	-0.1390	0.1280×10^{-3}	0.1494	0.3685×10^{-3}
6	-0.0046	0.1659×10^{-3}	-0.1681	0.3331×10^{-3}
7	-0.0944	0.1481×10^{-3}	-0.0435	0.2715×10^{-3}
8	-0.1051	0.0781×10^{-3}	-0.0092	0.2464×10^{-3}
9	-0.0209	0.0516×10^{-3}	-0.1027	0.1864×10^{-3}
10	0.0486	0.1129×10^{-3}	0.2044	0.1864×10^{-3}

TABLE 22

Forecasts of Peak Power and Energy Demand by Time-Series Analysis

95% confidence level.

YEAR	Peak Power Forecasts			Energy Demand Forecasts		
	Mean Value	Upper control limit	Lower control limit	Mean Value	Upper control limit	Lower control limit
1976	10160	11780	8831	24550	27630	22390
1977	10990	12670	8710	26980	29680	22960
1978	12220	13710	8790	29560	31840	23600
1979	13650	14860	8979	32660	34200	24380
1980	15350	15140	8642	35990	36730	25410
1981	17300	17580	8732	39540	39620	26360
1982	19140	19590	8872	42460	43550	27540
1983	20890	22230	9057	45600	47860	28840
1984	23330	25230	9311	48980	52600	30270
1985	24890	28640	9594	52600	57680	31770
1986	27100	32510	9931	56490	63240	33810
1987	29650	36990	10300	50670	69180	37180
1988	32280	41780	10720	65310	75680	39540
1989	35400	47420	11190	67010	82990	39170
1990	39540	53700	11720	75500	90360	41400
1991	42170	60670	12250	80910	99080	43750
1992	46230	68710	12880	86900	108900	46340
1993	50510	77800	13550	93510	117200	48980
1994	54990	87700	14290	100500	127900	51880
1995	59980	99080	15070	107900	139300	54950
1996	65610	101700	15920	115900	151200	58340
1997	71610	125900	16830	124500	165200	61800
1998	71860	129100	17820	133700	179900	65460
1999	85510	159200	18800	143500	195900	104700
2000	93330	179100	20000	154200	212800	116900

6.5 DISCUSSIONS OF RESULTS OBTAINED FOR FORECASTS BY USING TIME-SERIES ANALYSIS

ARIMA models of order (2, 1, 0) with differing parameter values have been used for forecasting peak power and energy demand. The models identified for the processes generating these demand series turn out to be non-stationary. Non-stationarities of the demand series are confirmed by the existence of a trend component which is an increasing function of time. We observe large differences in the magnitudes of forecasts obtained by the econometric methodology and time-series analysis methodology. It is very difficult to provide specific reasons for this wide differences. A possible reason might be that the parameters of the ARIMA models have undergone structural changes over the time span of observations. The time-series analysis methodology suggested in this dissertation will not provide accurate forecasts in the eventuality of time variant parameters. Further from the length of the record available it is difficult to draw any conclusions regarding the nonstationarity of parameters. Adequate length of records are essential to estimate a reliable model of the process generating the time-series. In our study the length of the time-series is not adequate for a realistic and accurate representation of the process. The possibility of errors of measurement of data is not ruled out.

In the nature of things it is difficult to realise the assumptions, based on which projections have been made. However we have sought on the basis of data available some independent estimates of likely levels of demand. Generally it has been found that estimates resulting from those based on macro economic variables provide reliable forecasts. It is expected that as more and more data becomes available comparison of actual values with forecats will enable a meaningful revision of forecasts. We are reasonably satisfied that.unless there is a development of a kind of an unexpected nature forecasts may be treated as adequate and realistic. To have quantitative informations on the risks involved with plans based on these forecasts we have added the probabilistic dimension to forecasting. It needs to be pointed out that it is not possible to have the same measures of confidence on the long term forecasts beyond 1990. The experience in the next few years will provide a clear indication of the direction in which forecasts have to be revised.

The demand for electricity depends on many factors such as future pattern of economic growth, demand for the output of electricity using commodities. In a country like India which has a central planning system, in drawing up the plans factors such as regional development, intersectional relationships, imports, exports and social welfare are given due weightage and final targets are determined based on these considerations. However, it should

recognised that while plans represent desired levels of economic development in accordance with desired social objectives their execution depends on a variety of factors. Hence forecasts based on planned and anticipated growth rates may not at all be realised due to improper implementations.

Forecasting of electricity demand, in its truer meaning and form involves substantially more than the application of ratios, formulae and historical relationships. it is very much a venture into the unknown, a journey into the land of 'if', 'probably' and 'perhaps'. Any study of the future requires exercise of judgement. However, judgement is a much abused term. At its worst it is an invitation to whims and conjecture rather than an adjunct to sound rigorous thinking. Much of what may pass for judgement is in truth relies heavily on a sound methodological frame work. Forecasting being both a science and art, scientific tools and techniques are as essential as judgement and insight.

Epilogue

After studying the material presented in previous sections the reader is clearly justified in saying "this was all somewhat informative, but what are the real demand projections?" The answer to the implied criticism in the querry must be that we cannot offer definite demand

functions, that electricity demand functions are evolutionary variables which must be re-examined and changed when important determining factors are changed, and that the determining factors are made up of an ill defined interplay of physical, economical, social and political forces. Electricity demands are time dependent functions that reflect the status of our technology, our economic well-being, our social system, our leisure time habits and existing political realities. They will change with time.

CHAPTER VII

PLANNING FOR CAPACITY EXPANSION OF ELECTRIC POWER SYSTEM - A LITERATURE SURVEY

The literature on planning investments in electric power systems to satisfy future demands covers a fairly broad spectrum and is both qualitative and quantitative. In this chapter we present a brief review of the literature in the area of planning for capacity expansion with special emphasis on electric power systems.

A principal difficulty in determining minimal cost optimal strategy for capacity expansion is that the investment costs for capacity increment typically are subject to significant economies of scale. Mathematically this means that the investment cost functions for capacity expansion are non-convex. Thus the determination of a minimal cost expansion strategy is likely to entail the minimisation of non-convex function and we are faced with the problem of distinguishing the true global minimum from possible local minima.

Even the static location models, in which demands are fixed for a given point of time and it is desired to find plant sizes and locations to minimise total investment, operatings and distributing costs for meeting this

demand suffer from the problem. Most of the models of traditional location theory are of this form. A survey of the literature of such models is given by Bos (237) For the case with demand of uniform intensity over an infinite plane surface and a cost function composed of a fixed charge plus cost proportional to the size of expansion, he shows that average cost per unit of demand may be transformed to a convex function which can be solved directly. If market boundaries are finite and demand are concentrated however this simplification is ruled out. Baumol and Wolfe (238) developed a simple mathematical programming model for warehouse capacity problem with a finite number of demand points and locations which as they pointed out would find only local optima. Kuehn and Hamburger (239) demonstrated that such local optima might be poor approximation to the global optimum and developed a simple heuristic method to find reasonably good solutions for location problem. Other heuristic methods have been developed by Cooper (240,241), Feldman, Lehrer and Ray (242).

The static location problem has been formulated as an integer programming problem with zero-one integer variables corresponding to decision to not construct or construct expansions at various locations. For small problems solutions have been found by the method of

complete enumeration by Vietorisz and Manne (243). Approximate solution to larger problems have been obtained by the "one-point move" algorithm due to Manne (244). Effromson and Ray (245) suggested a promising approach for solving this specialised problem by use of branch and bound integer programming methods. The case of uncertainity in demand for static location problem has been studied by Gregory (246).

The problem of capacity expansion over time for a single location has been studied by Chenery (247), Mc Dowell (248), Manne (249), Coleman and York (250). Manne's work also considers a special case of probabilistic demand growth. Veinott and Wagner (251) have observed the mathematical equivalence of capacity expansion problem with other well known problems like determination of economic lot size inventory decision and equipment replacement policies. The single location capacity models is closely related to the inventory model of Whitin (252) and Hadley & Whitin (253). A dynamic programming approach for single location problem with finite time horizon and arbitrary increasing demands is given in Manne and Veinott (254).

The general problem of planning capacity expansion for several locations with demand growing over time has been addressed by Ghosh (255) in case of Cement industry in India. He did not include the complicacies

of economies-of-scale in investment cost, and solved the problem by linear programming, transportation model. Application of linear programming to capacity planning in various industries have been summarised by Ward (256). The additional element of economies-of-scale in investment costs for expansion is included in the models developed by Manne (257). For the case of two producing locations, Manne uses a simple two-phase cycle model in which the time interval between expansion is constant and identical for both locations. For the case of several locations he developed a heuristic method which assumes a constant cycle time interval for expansion at individual locations. However, the model permits the length of the cycle to vary among locations, provided that each such interval is an integral divisor of some longer period called major cycle. The problem is formulated as an integer programming model. The solution method utilizes heuristics and does not guarantee optimal solution. Manne (257) applied this method to three industries in India. The industries considered were Cement with ten locations, Caustic Soda and Fertilizers with fifteen locations each.

Another integer programming formulation coupled with the use of a branch and bound method has been used in investment planning study for the Brazilian Steel industry by Kendrick (258). Kendrick's model is a finite horizon model and is quite comprehensive.

The dynamic multilocation capacity planning problem is also closely related to the multiproduct multi-facility inventory model of Zangwill (259), with capacity increments at different locations corresponding to orders for different products or the same product at different facilities.

Sreedharan and Wein (260) have proposed a continuous time model with several types of plants. Their emphasis is on the optimal timing of several candidate sequences of plant installations. Erlenkotter (261) considers a model in which the type of plants are determined. by their locations. He seeks the optimal amount and timing of capacity expansions over a finite horizon. In his paper the system operating cost is the cost of transporting the product of plants to the points of demand. He also discusses a stationary planning model with an infinite horizon.

In planning the expansion of electric power systems there are two popular types of models : simulation and programming. Galloway et. al. (262) have discussed a model to assess the stochastic variations in the available capacity. Giguet is one of the earliest investigators to deal with the problem of capacity expansion of electric power systems. The main feature of the model is

to study the relative profitability of a particular plant in relation to a reference thermal plant. All plants which are considered to be potential plants are ranked on the basis of their relative profitability. The plants which have the highest profitability are selected until the total demand is satisfied. This method is satisfactory as long as various plants do not interact among themselves and there are no economies-of-scale involved. The cut-off method of Giguet has been an acceptable methodology till 1954.

Some investigation into the area of planning the capacity expansions of power systems have dealt with the classical project by project engineering, economic and political trade-off between hydel, steam and nuclear power generation. Jacoby (263) gives a good description of the methods presently used in practice. Most of the methods have either used linear programming or simulation.

The French Federal Power Commission has done considerable work in using linear programming in numerous studies of investment planning in electric power industry. Masse and Bessier (264), Masse and Gibrat (265) are some of the important contributors.

The model developed by Dantzig (266) is in the same lines as Masse and Gibrat's model except that it

includes the time dimension. In this model element of power transmission are not considered.

More recently the Electricite' de, France (EDF) published the investment '85 model (267) consisting of 159 variables and 53 constraints. The convex quadratic cost function and the linear constraints assumed in the model uses the Wolfe's reduced gradient optimisation routine. This routine makes use of the fact that the constraints are linear, and hence employs partially the simplex procedure of linear programming.

Lack (268) formulated and solved a linear dynamic model of the diversity problem in which the problem of determining the trade-off between adding more generating stations at one of the two inter-connected load centres or adding more transmission capacity was considered. Chen et. al (269), and Sautter (270) formulated and solved the same diversity problem by using linear programming models.

Sequential probabilistic linear programming was employed by Manne to calculate the optimal electricity plant mix decision during the decade 1980's given the uncertainities of the date of availability of breeder reactor. The model allowed for the possibility that future uranium resources scarcities might lead to an increase in electricity prices and hence a reduction in projected demand.

Narasimhan's (271) linear programming model considered the transmission of power between generating and load centres along with the capacity planning problem of determination of size and location. Capacities and energy capacities during each season of new plants were taken as the variables of the model. The objective function was sum of cost of total energy for each plant which included fixed and running costs. Constraints of the model were upper bounds on capacities and energy balance of the entire system. In this model transmission of power was taken to incur only fixed cost of transmission equipments and transmission losses were ignored.

In a dissertation by Gosai (272) the L.P. model has been used to determine location, type and capacities of power plants to be added to an existing system. The concept of product mix and integrated grid system was incorporated in this model. The objective function for his model minimised the annual amortised cost, annual generation cost and annual transmission cost. The constraints took into account the power losses in the system.

Tikaria (273) has developed a nonlinear programming model of planning for capacity expansion in a grid. The objective was to minimise the sum of transmission loss cost, annual amortised costs of plants and running cost. The model formulated has a quadratic

objective function with linear constraints, which has been solved by using sequential unconstrained minimisation algorithm.

In England Dale (274) suggested the use of dynamic programming for the problem of determination of size and time of installation of power plants, neglecting the location aspect of the problem.

Integer linear programming formulation has been used by Okada and Kawai (275), to select sites, type and capacities of new plants in order to meet the specified demand with minimum cost. A stepwise cost capacity characteristic has been considered in this formulation which is more realistic than a linear characteristic, which has been assumed in all linear programming models. But power losses in the system and upper and lower bounds on capacities of plants have not been considered.

Various books and articles have been published on the system simulation approach to the whole problem of planning size and location and time phasing of future series of power plants. Most of the publications present fairly complex models but use no optimisation. Sensitivity analysis is used in most cases to obtain the various good feasible solutions. Marglin (276), Nelson (277) and Bary (278) present very comprehensive description of such

aspects of the problem as pricing, cost to serve, interconnection between systems etc., but fail to give a computationally efficient approach that will include all the economic aspects of the problem.

Cazalett (279) presented an extension of the Everett's generalised lagrange multiplier approach to unconstrained minimisation problem. However only the strategic plant capacity sub-problem was analysed. The tactical variables i.e., those related to operation by various plants in the system were approximated by using a hypothetical operating policy that loads plants in order of efficiency. A fairly complex simulation oriented towards Nuclear plant feasibility analysis was published by S.R.I. (280) in conjunction with Mexican Federal Power Commission (using Cazallet's concepts).

Jacoby (263) presented a very comprehensive simulation model which analytically included many of the ideas of Baldwin (281,282)and Dary (278). Although the approach included operation, reliability, environmental, social value and evaluation submodels it did not include any optimisation procedure to determine the best schedule of plant installation.

The survey of the reported literature reveals that mathematical programming models have been extensively used for solving the problem of capacity expansion of electric power systems. However the models proposed, have tackled the problem of capacity expansion and operational planning independently and seperately, with capacities as continuous variables. We rarely find any evidence of a methodology which obtains an integrated solution of the overall capacity expansion problem incorporating the capacity expansion aspects and operational planning aspects simultaneously. Further most models proposed, treat the capacities as continuous variables, where is in a real life situation capacity increments take place only in discrete steps, due to standatd in force, availability of specific sizes of plants in the market, and manufacturing considerations of power plant equipment. A capacity expansion program which does not consider the feasibilities of expansion at a particular location of a given type and size of plant in a given period, is unlikely to provide a pragmatic solution to the capacity expansion problem. If it were possible to draw up a complete list of all feasible power plant developments in a region the problem would reduce to choosing the best combination and sequence of power plants to meet the projected power demand over the planning horizon. But such a list of feasible development multiplies as soon as several

locations, sizes and construction periods and type of projects, (such as hydel, thermal and nuclear plants) are taken into consideration. Because of the problem of dimensionality most studies up to now have used a screening type of studies to obtain a few possible power plant sequence, but have failed to obtain the best solution.

In this dissertation general models that include the main aspects of electrical power system capacity expansion problem are proposed. The approach is neither purely simulation nor analytical oriented. Model simplification has been made in the interest of obtaining numerical results but such realism as transmission lines, operating policies are included. Gradual improving expansion policies are obtained and each time their feasibility with respect to satisfying both capacity and energy are tested.

The complexities of power system planning is due in part to the technical interaction between generating plants and economic impact of their interactions. The economic importance of fuel costs require the analyst to study the coordinated operation of the individual plants as well as capacity expansion of the system. The large number of possible combinations of individual plants with a realistic planning period has been the major obstacle in the use of optimisation techniques for planning the capacity expansion

of electric power system. Model decomposition and iterative concepts prove very useful in partly resolving this combinatorial problem.

The purpose of system modelling is to construct a mathematical model such that response of both the real system and mathematical model to the same input are almost identical. But a mathematical model cannot include all the detailed aspects of a large scale electrical power system in a developing region. Thus the aim must be to represent as realistically as possible only those aspects of the problem which are important. The theory and applications of the mathematical model of a complex system are still in an early stage of development. A complex realistic model with many locally good solutions ideally requires an efficient optimisation routine that will find the globally optimum solution. However, this is not always feasible and the problem is again one of determining the best trade-off between efficiency and cost of optimisation techniques.

With the aforementioned considerations an attempt is made in this dissertation to suggest a methodology for determining the minimum cost expansion of capacities of an electric power system in a region incorporating the capacity expansion aspects as well as operational planning aspects in an integrated manner. The suggested approach

systematically searches for an optimal solution.among feasible alternatives. The iterative use of capacity expansion planning models together with that of the operational planning model is is what distinguishes our methodology from other mathematical models. In Chapter VIII we present the mathematical formulation of the capacity expansion problem and the operational planning problem and their solution methodologies.

CHAPTER VIII

PROBLEM FORMULATION AND SOLUTION METHODOLOGY

This chapter is devoted to the problem of planning for capacity expansion of an electric power system. Section 8.1 gives a statement of the overall capacity expansion problem and the need for a decomposition method to solve this problem. The overall capacity expansion problem is treated as a combination of two sub-problems, viz. capacity expansion sub-problem and operational planning sub-problem. Section 8.2 presents the mathematical model of the capacity expansion sub-problem. In section 8.3 we provide the solution methodology for the capacity expansion sub-problem. The formulation of the operational planning sub-problem and its solution methodology is presented in section 8.4.

8.1 PROBLEM STATEMENT

The purpose of this study is to analyse using an integrated approach the complex problem of electricity supply, loads, and economic constraints in planning the best location, size (capacity), type, and time of installation of a series of power plants in a region to meet the future growing demands for electricity.

In the analysis presented here, the overall capacity planning problem is decomposed into two parts. They are referred to as the capacity expansion sub-problem and the operational planning sub-problem.

1. Capacity Expansion Sub-problem - This problem involves the determination of a minimum cost (discounted) sequence of future plant installations such that the total system which might consist of a mix of hydel, thermal and nuclear plants has at all times a total capacity sufficiently large to meet the peak load demand plus reserve requirements. This problem is referred to as the *strategic* problem.

2. Operational Planning Sub-problem - This problem involves the determination of the discounted minimum cost of operation of such a sequence of power plant installations. The sub-problem is referred to as the *tactical* problem.

From the operation planning sub-problem the production and other system characteristics are calculated. The parameter values obtained from the solution of the operational planning sub-problem are then compared with parameter values assumed in capacity expansion sub-problem. If there is a difference in assumption of average operation in (1) and the calculated operation results in (2) the assumed paramater values, used in the capacity

expansion sub-problems are altered. This feed back is repeated until the expansion sequence in (1) has no further improvement. The same approach has to be carried out for different demand schedules. By the methodology of decomposing the problem into two parts, the primary purpose is to find a quasi-optimal solution to the overall large scale problem.

The <u>strategic</u> problem corresponds to the capacity expansion sub-problem and is the portion of the procedure that emphasises the capacity installations (MW) aspects, i.e., determination of quantities of different types of equipments to be constructed.

The <u>tactical</u> or operational planning sub-problem emphasises the operational aspect of the plants chosen by the strategic calculations. In particular the operational planning problem determines the minimum variable cost operating policy to satisfy the projected energy demand, (thus providing the feasibility of the current expansion policy).

The feed-back between the two problems gradually improves the fuel cost estimates so that a true minimum of fixed plus variable costs are ultimately obtained for the overall capacity expansion problem. If the objective is the minimisation of fixed costs only, rather than fixed

plus variable costs, only the capacity expansion problem need be solved, since investment cost for each alternative plant is known. However, the variable cost of each plant are not known as they depend on the alternatives chosen by the capacity expansion sub-problem. The variable costs would be known if these plants operated in isolation from each other, i.e., without any systems co-ordination.

The above methodology is based on the observation that tentatively fixing the values of the plant energy variables renders the problem simpler, which is solvable in this case by an integer programming algorithm. These plant energy variables (which are tentatively fixed) are referred to as the complicating variables. Benders (283) was the first to formalise a decomposition procedure for solving such problems. By fixing the complicating variables given problem was reduced to a linear programming problem (parametrised by the complicating variables).

The general formulation of Bender's semi-linear problem is given by

$$\underset{\substack{X \quad 0 \\ y \quad Y}}{\text{Maximise}} \left[C^T x + f\,(Y) \right] \qquad (8.1)$$

such that

$$A\,x + F\,(y) - b \geqslant 0 \qquad .1$$

where

X, Y are vectors

A and F are matrices

f is a scalar function.

The code proposed by Bender (283) for finding the optimal value of the complicating variables, involves two steps. The steps are:

(1) Manipulate the above formulation of the problem by projection of space of Y variables (Refer (284)) to yield

$$\underset{y}{\text{Maximise}} \left[v\,(y) \right]$$

such that

$$y \in Y \cap V \qquad (8.2)$$

where $v\,(y) = \underset{x}{\text{Supremum}} \left[C^t x + f\,(y) \right]$

such that

$$A\,x + F\,(y) - b \geqslant 0 \qquad (8.3)$$

and

$$V \equiv \left\{ y : A\,x + F\,(y) - b \quad 0 \text{ for some } x \in X \right\}$$

So the projected problem equivalent to (8.1) is that an optimal solution y^* of (8.2) readily yields an optimal solution X^*, Y^* of (8.1).

(2) Since v, and V are only known implicitly solve (8.2) by a cutting plane method that builds up a tangential approximation to v and V. Linear programming duality theory is employed to derive the family of cuts characterising their representation.

The electrical power planning problem in this dissertation can be formulated in an integrated form as

$$\text{Max } \left[- C^T (x) - f (y) \right]$$

$$\text{s.t } P (x, y) \geqslant 0$$

where

x = vector of 0, or 1 variables that represents decision to build or not to build plants and transmission lines.

y = vector of annual plant energy productions.

The vector of constraints $P (x, y) \geqslant 0$ includes satisfaction of capacity (MW) constraints, i.e., $P_1 (x) \geqslant 0$ and satisfying the energy constraints i.e., $P_2 (x, y) \geqslant 0$.

By using the non-linear convex duality theory Geoffrion (285) has recently extended Bender's approach to a broader class of problem in which the parametrised sub-problem need not be linear. A master program is developed that would direct the sequence to final choices of the complicating variables. The computational algorithm consists of iteratively solving (8.3) and a relaxed

master problem that ignores some of the original inequality constraints. The solution to the relaxed master programs is such that (8.3) is feasible.

The effectiveness of the above algorithm is limited however to a special class of problems such as variable factor programming problem (286) where the solution of the relaxed master problem do not too often lie outside V.

Here a direct non-linear programming technique can be used for the operational planning sub-problem due to the availability of solution algorithm for capacity expansion sub-problem and operational planning sub-problem. One of the advantage of this method is that by analysing the operation planning problem results at every iteration, one obtains useful insight and information. This can be used to modify the capacity expansion sub-problem, at the next iteration.

8.2 FORMULATION OF THE CAPACITY EXPANSION SUB-PROBLEM

Before formulating the model we discuss in brief about the necessity of a coordinated system design in the following paragraphs.

Various types of power generating plants available today, have comparative advantages and disadvantages depending on several factors. Therefore a coordinated

system design and operation of the different power facilities is necessary to minimise the total cost of meeting electricity power and energy demand.

A planner is not only concerned with the coordination between various type of plants that are devised but also with coordination on a multiregional basis, between different interconnected systems. The problem of coordinating an existing system in a region is different from that of planning the coordination of future expanding systems. To properly solve the later problem it is necessary to find out the best future system which consists of the existing system plus sequence of plant installations throughout the planning horizon T, and then to carry out a system operation study which will determine the best annual operating policy of the entire plant mix. The operating rules will be changing as the systems gradually expands over time T. This problem is called the operational planning problem, to be dealt in section 8.4 However, the minimum cost sequence of plant installations will be known only if annual fuel cost of various feasible sequences are known, and this requires knowledge of system load factor, and the best operating procedure. The problem is apparently circular. This is the reason why this study suggests the adoption of a two step methodology. presented in Figure 8.1.

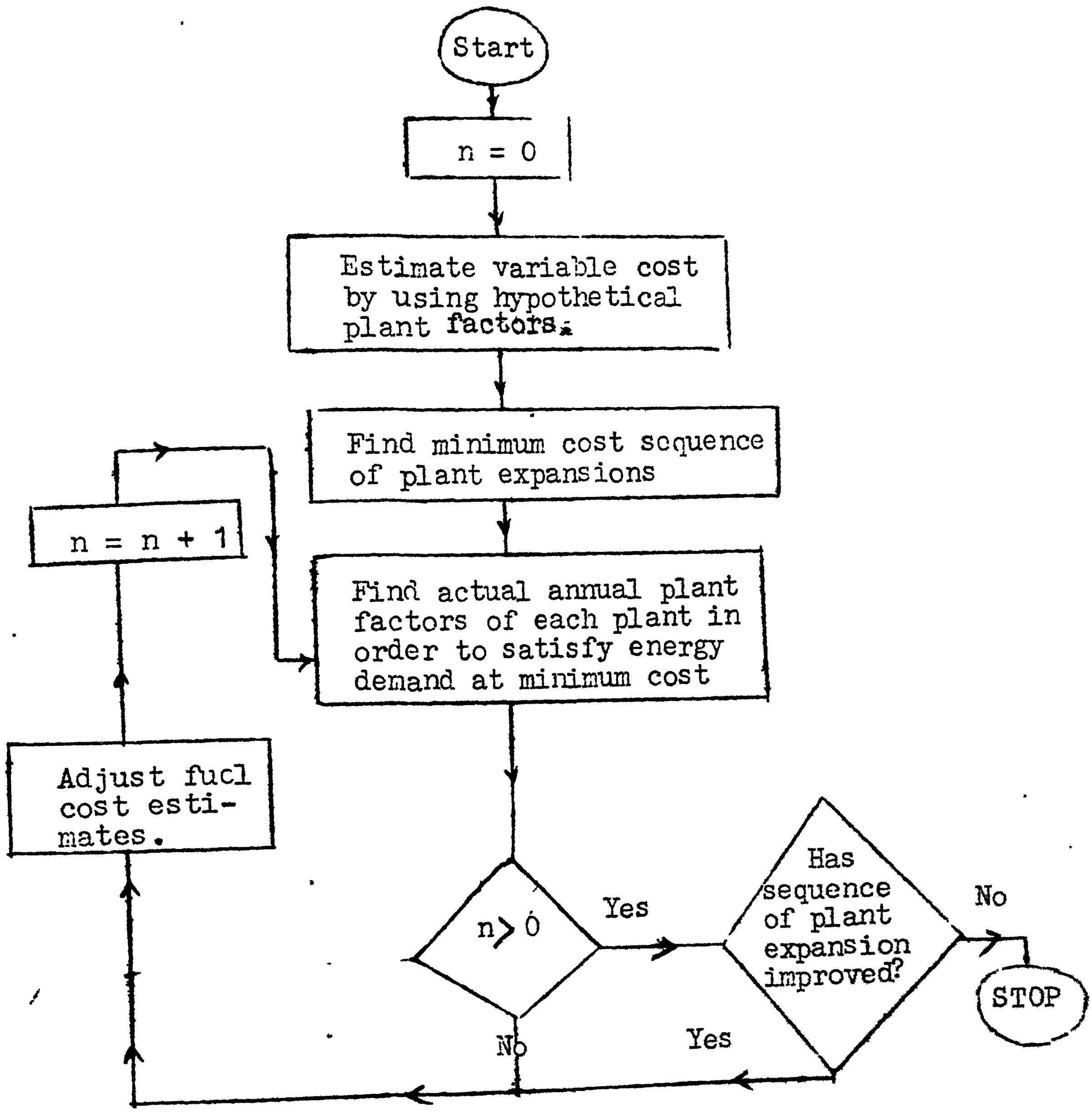


FIG. 8.1 : Diagrammatic representation of the two-step methodology.

The capacity expansion sub-problems uses the criteria of minimisation over T years, the discounted fixed cost plus average variable costs. The variable costs are mainly fuel cost and depend on average plant factors of the plants in the system. The decision variables in this problem include the capacity or size (MW) of different types of plants, to be built in a number of available sites over a planning horizon of T years.

One of the distinguishing characteristics of the investment, operating and maintenance costs in the electricity industry are that they are non-linear in nature. In addition, the problem of choosing minimum cost expansion is an integer valued problem, since only standard size equipments are available, for capacity expansions. Also, the problem is a fairly large enumerative effort as soon as a few alternative capacity increments for each type of plant are introduced.

The general integer programming model differs from the linear programming formulation in that the later uses continuous variables. To use the implicit enumeration solution technique an even more restrictive constraint is required, that is the decision variables must have only one of the two discrete values, zero or one.

The general 0 - 1 integer programming problem is of the form

$j = 1, 2 \ldots N_3$ - are the alternative load centres considered throughout the region.

$t = 1, 2 \ldots N_1$ - are the time period into which the total planning horizon T has been devided.

8.2.2 Model Formulation

The formulation of the capacity expansion sub-problem is presented below.

$$\text{Minimise} \sum_{t=1}^{N_1} \left\{ \sum_{i=1}^{N_1} \sum_{j=1}^{N_3} \left[C_{ijt}\, x_{ijt} + K_{i,j1-j2,t} \cdot Z_{i,j1-j2,t} \right] \right\} \qquad (8.5)$$

such that

$$\begin{array}{lll} x_{ijt} = 0 \text{ or } 1 & & t = 1, \ldots N_1 \\ y_{ijt} = 0 \text{ or } 1 & \text{for} & i = 1, \ldots N_2 \\ z_{ijt} = 0 \text{ or } 1 & & j = 1, \ldots N_3 \end{array} \qquad (8.6)$$

$$A^p_{jo} - \sum_{t=1}^{t} L^p_{jt} + \sum_{t=1}^{t} \sum_{i} R_{ij} \cdot x_{ijt} + \sum_{j} \sum_{i} R^p_{ijt} \cdot y_{ijt} \geqslant D^p_{jt} \qquad (8.7)$$

$$A^b_{jo} - \sum_{t=1}^{t} + L^b_{jt} + \sum_{t=1}^{t} \sum_{i} R_{ij} \cdot x_{ijt} + \sum_{j} \sum_{i} R^b_{ijt} \cdot y_{ijt} \geqslant D^b_{jt} \qquad (8.8)$$

for all $t = 1, \ldots N_1$

$j1 = 1, \ldots N_3$

$j2 = 1, \ldots N_3$

$S_{i,j1-j2}$ - The MW transmission capacity of the i^{th} transmission line alternative between load centres j1 and j2.

$S_{o,j1-j2}$ - The MW transmission capability of transmission lines between centres j1 and j2 at the beginning of planning horizon (T = 0).

$\beta^{t}_{i,j1-j2}$ - (1 + A) where A = percent heat and other losses in the i^{th} transmission line between j1 and j2 during period t.

A^{b}_{jo}, A^{p}_{jo} - Total base and peaking capacity respectively, of the j^{th} region at start of planning period (t = 0).

D^{b}_{jt}, D^{p}_{jt} - Total base and peaking capacity requirement respectively in the j^{th} region at the start of t^{th} period (includes a certain reserve requirement).

L^{b}_{jt}, L^{p}_{jt} - Base load and peaking load plants respectively, retired in the j^{th} region at the start of the t^{th} period.

$i = 1, 2 \ldots N_2$ - Represents the alternative plant types (size and capacity) available and alternative transmission line (size and capacity) available.

Definitions of Terminology

X_{ijt} - 0 - 1 decision variables that chooses the i^{th} type (representing size and type) during the t^{th} period at the j^{th} location.

Y_{ijt} - 0 - 1 decision variables that chooses the export of i^{th} type (representing type and amount) from the j^{th} region during period t.

$Z_{i,j1-j2,t}$ - 0 - 1 decision variables that chooses the i^{th} transmission line alternative between load centres j1 and j2 during period t.

C_{ijt} - The sume of discounted fixed plus estimated variable plant investment, operating and maintenance costs incurred in choosing the i^{th} alternative plant in the j^{th} location during period t.

$K_{i,j1-j2,t}$ - The sum of discounted fixed plus variable transmission facility, investment, operating and maintenance costs incurred in choosing the i^{th} transmission line alternative between load centres j1 and j2 during period t.

R_{ij} - Plant capacity installation increment allowed by the X_{ij} alternative independent of time period t.

R_{ijt} - The import or export capacity of type and size i at j^{th} location, during period t.

$$\underset{x_j}{\text{Minimise}} \quad \sum_{j=1}^{n} C_j \, x_j \tag{8.4}$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij} \cdot x_j \leq b_i$$

$$i = 1, 2, \ldots m$$

$$x_j = 0 \text{ or } 1 \qquad j = 1, 2, \ldots n$$

In this section a zero - one integer programming model for the capacity expansion sub-problem is presented for determining the best strategy for capacity expansion.

8.2.1 Model Description

The capacity expansion problem will be formulated as an integer programming model. The decision variable is to choose an installation policy (X_{ijt} , Y_{ijt}) and export-import policy (Z_{ijt}) for the expansion of electrical system in a region. By a policy we imply a complete list of plants and transmission facilities to be installed and quantities of electricity to be exported or imported between different sub-regions over the planning period of the analysis. The planning period T is divided into n number of 5 year plans during which a decision to build or not to build (X_{ijt} = 0 or 1 and Z_{ijt} = 0, or 1) and to import or not to import (Y_{ijt} = 0 or 1) is made.

and

$$\beta^{t}_{i,j1-j2} \cdot \left\{ \underset{j}{\text{Maximum}} \left[R_{ijt} \cdot y_{ijt} \right] \right\} \leqslant \sum_{t=1}^{t} \sum_{i} \left\{ S_{i,j1-j2,t} \cdot z_{i,j1-j2,t} \right\} + S_{o,j1-j2} \qquad (8.9)$$

Equation (8.5) is the objective function which minimises the sum of discounted fixed costs and total estimated variable costs and operational and maintenance costs for all plants and transmission lines. Equation (8.6) is the zero-one restriction required for the use of enumeration as a method of solution. Equations (8.7) and (8.8) are the restrictions necessary to keep the total available peaking and base load capacity at the j^{th} region in every period t at least as large as the required peaking and base load capacity D^{p}_{jt}, D^{b}_{jt}, which is obtained by projecting the present load requirements in the region. D^{p}_{jt}, D^{b}_{jt} include certain reserve capacity which is necessary for meeting any expected plant failures. Because of transmission losses, R_{ijt} is actually a reduced import in equations (8.7) and (8.8). Equation (8.9) restricts the capacity of the transmission lines between load centres. For two particular load centres j1 and j2 the total transmission capacity during period t must be greater than or equal to the maximum of the capacities imported or exported between j1 and j2, increased by the amount of MW losses expected for the transmission lines.

To calculate the K8s and C's of the objective function it is first necessary to determine the type of plants and transmission lines that can be built in a region. Knowledge of various type of power plants and transmission lines available today and in future are necessary to make estimation of these costs. In the present capacity expansion model total variable costs for various type of plants alternatives are estimated by assuming approximate average plant factors, since the details of the co-ordinated system operation is not known until the sequence of installations are chosen.

In general, for most regions there will be a large number of alternative power plants(types and size) and sites (i and j of the model). The planning agencies usually carry out site screening studies to limit the number of choices on site and sizes and type as much as possible. Of course it may so happen that constraints also determine the particular site and type of plant to be built in the next few years. In order to solve the capacity expansion problem formulated in this section it is necessary to carry out such screening studies based on several factors.

8.3 SOLUTION METHODOLOGY FOR THE CAPACITY EXPANSION SUB-PROBLEM

The implicit emmeration zero-one integer variable algorithm proposed by Balas (   ) has been used for solving the integer programming model of capacity expansion

sub-problem. The algorithm uses heuristic tests suggested by Holcombe (287), because of its high efficiency and low cost relative to other integer optimisation routines like the branch and bounds method by Land and Doig (288), and statistical sampling method by Beiter and Sherman (289).

8.4 FORMULATION OF THE OPERATIONAL PLANNING SUB-PROBLEM

In section 8.2 the formulation of the capacity expansion sub-problem to meet future load schedules at minimum discounted costs were presented. The capacity expansion model determines the best sequence of future capacity installations in the region. In this section we present the long-term operational planning model.

The fuel costs are different due to the types of plant, economies-of-scale, technological improvement. The objective of the long term operational model is to meet energy requirement. over the planning horizon, at minimum operation cost, i.e., to optimally allocate load among available plants. The solution of the operational planning model will provide us the annual plant factors of the various power plants in the region such that total discounted costs are minimum.

The operational planning model is a non-linear programming model. The decision variables are to choose

the yearly generation of energy (MWH) at each of the j alternative plants in the region in each year over the planning horizon.

Definition of Terminologies

$t = 1, 2, \ldots T$ - represents the time subscript which refers to the number of years.

$i = 1, 2, \ldots N_1$ - refers to type and amount of energy generated, imported or exported..

$j = 1, 2, \ldots N_2$ - refers to location from which energy is being generated or exported.

d_{ijt} - Number of MWH to be produced during t by i^{th} (type and size) plant at j^{th} location. This is a continuous variable.

d^b_{ijt}, d^p_{ijt} - refers to base load energy and peaking energy.

e_{ijt} - the i^{th} amount of energy imported ($+ e_{ijt}$) or exported ($- e_{ijt}$). (These are constants determined by the import-export capacities chosen in the capacity expansion problem.)

e^p_{ijt}, e^b_{ijt} - refers to peak and base capacity.

E^b_{jt}, E^p_{jt} - refers to net base and peaking energy demand respectively in the j^{th} region during the t^{th} time period.

q_{ijt} - fuel price for i^{th} type of plant at the j^{th} location during period t.

$H_{ijt}(\sum_t d_{ijt})$ - The average net heat rate function of the i^{th} type of plant at j^{th} location during period t. This is a function of the generation history of the plant i.e., $_t\, d_{ijt}$

m_{ijt} - Variable operation and maintenance costs for the i^{th} type of plant at j^{th} location during period t.

P_{ijt} - Penalty costs for adverse effects (such as pollution) of the i^{th} type of plant, at the j^{th} location during period t.

G_{ijt} - The maximum total number of MWH that the i^{th} type of plant at the j^{th} location is allowed to generate during the time period t.

With the above terminologies, we specify the objective function as

$$\text{Minimise} \quad \sum_{t=1}^{T} \left\{ \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \left[H_{ijt} \left(\sum_t d_{ijt} \right) q_{ijt} \cdot d_{ijt} + (m_{ijt} + p_{ijt})\, d_{ijt} \right] \right\} \qquad (8.10)$$

such that

$$\sum_i d^b_{ijt} \geqslant E^b_{jt} \pm \sum_j \sum_i e^b_{ijt}$$

$$\sum_i d^p_{ijt} \quad E^p_{jt} \pm \sum_j \sum_i e^p_{ijt} \qquad (8.11)$$

for $i = 1, \ldots N_1$

$j = 1, \ldots N_2$

$t = 1, \ldots T$

$$d_{ijt} \leqslant G_{ijt}$$

for $i = 1, \ldots N_1$

$j = 1, \ldots N_2$ (8.12)

$t = 1, \ldots T$

Equation (8.10) is the objective function which is the minimisation of total discounted fuel costs, plant operation and maintenance costs plus penalty costs.

In section (8.2) the general model of the integrated electric power system was presented using the ideas of Bender's. In that formulation of the problem, the non-linear continuous function f (y) corresponds to the operational planning sub-problem here. The capacity expansion sub-problem described in section 8.3 is represented by the non-linear integer variable portion i.e., function C (x). The integrated Bender's decomposition formulation is a non-linear mixed integer programming problem where the complicating variables are identified with the integer

model. Farther into the future we project, the projections become more and more uncertain. Hence adaptive modelling is recommended. Results of the model are implemented for the immediate future (say five years for a planning horizon of thirty years) after which the model is modified or adapted to take into account changes that have taken place in the immediate and recent past. The problem is again solved and decision of the immediate future are implemented. Uncertainity of nuclear fuel costs calls for such an adaptive modelling.

The primary component of the steam fuel costs are directly proportional to amount of energy generated and are about 90% of the total costs of fuel. The secondary costs of fuel are attributed to spinning reserves, starting and stopping of plants. The secondary fuel burnt is a function of the net heat rate and hours of use, and constitutes approximately 10% of the total costs.

The nuclear fuel costs comprises of cost components like value of the fuel burned, fabrication costs, shipping costs for both irradiated and new fuel, chemical reprocessing of irradiated fuel and such other costs. Fuel burn up or irradiation level is given by MW days of heat generated per ton of uranium.

CHAPTER IX

RESULTS, DISCUSSIONS, CONCLUSIONS AND SCOPE FOR FURTHER RESEARCH

In this chapter we shall present the numerical results obtained for a case study by using the models developed in Chapter VIII for the capacity expansion planning sub-problem and operational planning sub-problem of an electric power system in a region. The sources of data used for this case study have been listed in Appendix A.

The planning time horizon for the capacity expansion sub-problem as well as the operational planning sub-problem has been assumed to be thirty years (1971 - 2000). For the capacity expansion sub-problem the planning horizon is divided into six time periods each of duration five years, i.e. 1971-76 corresponds to period 1, 1976 - 81 corresponds to period - 2 and so on. For the operational planning sub-problem the planning horizon has been divided into thirty annual plans each of duration one year. The demand projections have been given for five year periods. For calculating the future requirements of energy in the region a load factor of 70% has been assumed. Energy requirements for each period have been calculated by the following relationship.

Energy requirements = 0.7 x 8760 x project capacity demand. Base demand has been obtained from the relationships given below.

Base demand (MW) = 0.5 x Maximum demand (MW)

Base energy demand = Base capacity x 8760 (MWH)

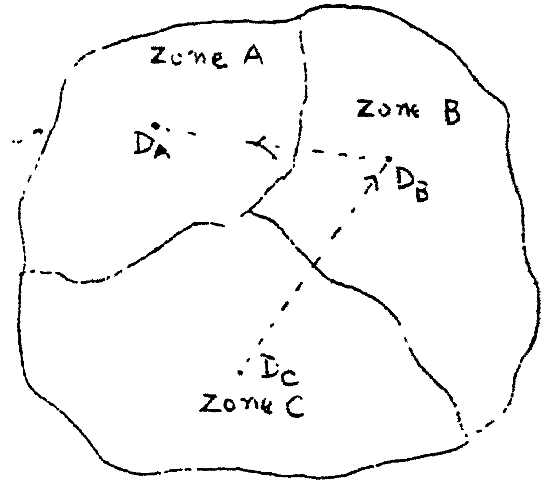
Peak energy demand = Total energy demand - Base energy demand.

The life span for steam plants and hydel plants have been assumed to thirtyfive and forty years respectively. For calculating the total capacity demand for which planning has to be done, the reserve capacity is taken as fifteen percent of expected peak demand.

9.1 THE MODEL FOR THE SYSTEM USED FOR THE CASE STUDY

The entire region has been devided into three zones A, B, C for the purposes of this study, based on the similarity of the characteristics of demand in each zone. As explained earlier, both the capacity expansion and the operational planning models assumed that the region being supplied by the electric power system under study has demand centres between which long distance transmission takes place.

The fixed cost of alternative projects not located at these load centres will be increased by the cost of transmission lines that would have to be built from the plant to supply electricity at load centres.



This model utilises the results of a screening study made by the planning agencies to determine the appropriate alternative projects in the entire region. The results of this screening study supply the basic input data to the models proposed in this dissertation and hence the initial screening phase is vitally important. If the resulting expansion alternatives to be supplied to the model are not good, the methodology suggested in this dissertation can only find the best among the alternative expansion policies which may not be optimal.

A selected sample of the results of such a screening study have been given in Table - 23. For the entire region 97 alternative power plants, 24 transmission line alternatives, and 44 importation - exportation alternatives have been chosen as feasible alternatives, to be supplied as input data to the model.

The plant factors have been mainly used to calculate costs and do not represent initial operating policies.

TABLE 23.

A sample of alternative plants, and transmission lines and importation exportation alternatives for the region obtained from screening studies

(i) Alternative Plants

Zone	Name of project (location)	Plant type and size	Probable installation period	Symbol used for this alternative
A	Guyamas - I	100 MW Steam Plant	1971 - 75	X_1
A	Guyamas - I	25 MW Steam Plant	1976 - 80	X_5
⋮	⋮			⋮
A	Baja	30 MW Hydel Plant	1981 - 85, 1986 - 90, 1991 - 96, 1996 - 2000	⋮
B	Obregon -I	30 MW Gas Turbine	1971 - 75	⋮
⋮	⋮	⋮	⋮	⋮
B	Obregon III	120 MW Gas Turbine	1976 - 80, 1981 - 85, 1986 - 90, 1991 - 95, 1996 - 2000	⋮
C	Bacurata	20 MW Hydel	1971 - 75, 1976 - 80, 1981 - 85	⋮
⋮	⋮			
C	Machis - IV	200 MW Steam	1986 - 90, 1991 - 95, 1996 - 2000	X_{75}, X_{81}, X_{97}

(continued on next page)

TABLE 23 (Continued)

(ii) Importation alternatives from the screening studies

Symbol for each alternative	Import to zone	Amount of import (MW)	Probable importation period	Type
y_1	A	5	1971 - 75	P
y_2	A	10	1971 - 75	P
.	.			
.	.			
.	.			
y_{44}	A	150	1996 -2000	B
y_{45}	B	30	1971 - 75	B
.	.			
.	.			
.	.			
y_{78}	B	60	1996 -2000	B

(iii) Transmission line alternatives from screening studies

Type	Transmission line projects between Zones A and B					
	1971-75	1975-80	1981-85	1986-90	1991-95	1996-2000
S/C 220KV 795 ACSR	$Z_1^{+}=300^{*}$	$Z_3=300$	$Z_5=300$	$Z_7=300$	...	$Z_{11}=300$
S/C 345KV 2168 ACSR	$Z_2=700$	$Z_4=700$	$Z_6=700$	...	...	$Z_{12}=700$
	Transmission line projects between Zones B and C					
**S/C 220KV 795 ACSR	$Z_{13}=300$	$Z_{15}=300$	...	...	...	$Z_{23}=300$
S/C 345KV ++2168ACSR	$Z_{14}=700$	...	...	...	...	$Z_{24}=700$

* Assumed transmission capacity in MW
** S/C mean single circuit steel tower
\+ ACSR refers to conductor size
++ Z is the symbol (0 - 1) variables used for these alternatives.

Plant factors that have been used for calculating variable costs are as follows:

Hydel plants - 25%

Steam plants - 55%

Gas turbine plants - 15%

The capacity expansion model for the region considered is as follows:

$$\text{Minimise} \left\{ \sum_{i=1}^{16} \sum_{t}^{6} \left[C_{it}\, x_{it} \right] + \sum_{k=1}^{4} \sum_{t=1}^{6} \left[C_{kt} \cdot Z_{kt} \right] \right\}$$

where

C_{it} = discounted cost of plant i constructed during period t (includes operation costs).

x_{it} = 0 - 1 decision variables for plant i built during period t.

Z_{kt} = 0 - 1 decision variables for transmission lines k, built in period t.

C_{kt} = present worth cost of transmission line k, constructed in period t.

Demand constraints for Zone A

$$\sum_{i=1}^{3} S_i^U \; y_{i1} + \sum_{i=1}^{3} R_i^U \; x_{i1} + A_{1,0}^U - L_{11}^U \geq D_{1,1}^U$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\sum_{i=1}^{3} S_i^U \; y_{it} + \sum_{t=1}^{6} \sum_{i=1}^{3} R_i^U \, x_{it} + A_{1,0}^U - \sum_{t=1}^{6} L_{1t}^U \geq D_{6,}^U$$

Demand constraints for Zone B

$$\sum_{i=4}^{6} S_i^U y_{i1} + \sum_{i=4}^{6} R_i^U x_{i1} + A_{2,0}^U - L_{21}^U \geqslant D_{1,2}^U$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\sum_{i=4}^{6} S_i^U y_{it} + \sum_{t=1}^{6} \sum_{i=4}^{6} R_i^U x_{it} + A_{2,0} - \sum_{t=1}^{6} L_{2t}^U \geqslant D_{6,2}^U$$

Demand Constraints for Zone C

$$\sum_{i=7}^{9} S_i^U y_{i1} + \sum_{i=7}^{9} R_i^U x_{i1} + A_{30}^U - L_{31}^U \geqslant D_{13}^U$$

$$\vdots$$

$$\sum_{i=7}^{9} S_i^U y_{it} + \sum_{t=1}^{6} \sum_{i=7}^{9} R_i^U x_{it} + A_{30}^U - \sum_{t=1}^{6} L_{3t}^U \geqslant D_{6,3}^U$$

where

U = base, and peak

L_{nt} = various types of plant of type U retired in zone n at beginning of time period t

R_i^U = generation capacity i^{th} type of plant (of type U)

S_i^U = import export capacity of alternative i (of type U)

y_{it} = 0 - 1 decision variable for i^{th} import export alternative in period t

A_{jo}^U = Installed generation capacity of type U in region j at the start of planning horizon

D_{jn}^{U} = capacity demand of type u in region n during period j .

Transmission line constraints between Zone A and Zone B

$$L\,F_1\left[\sum_{u=\text{base or peak}} \sum_{i=1}^{4} S_i^{U}\, y_{i1} - \sum_{i=1}^{2} t_i\, Z_{i1}\right] \leq T_{1-2}$$

$$\vdots$$

$$L\,F_1\left[\sum_{u=\text{base or peak}} \sum_{i} S_i^{U}\, y_{it} - \sum_{t=1}^{6} \sum_{i=1}^{2} t_i\, Z_{it}\right] \leq T_{1-2}$$

Transmission line constraints between Zone B and Zone C

$$L\,F_2\left[\sum_{u=\text{base or peak}} \sum_{i} S_i^{U}\, y_{i1} - \sum_{i=2}^{4} t_i\, Z_{i1}\right] \leq T_{2-3}$$

$$\vdots$$

$$L\,F_2\left[\sum_{u=\text{base or peak}} \sum_{i} S_i^{U}\, y_{it} - \sum_{t=1}^{6} \sum_{i=2}^{4} t_i\, Z_{it}\right] \leq T_{2-3}$$

where

t_i = transmission capacity of transmission line i

T_{1-2} = transmission capacity between zone A and Zone B at start of planning horizon.

T_{2-3} = transmission capacity between zone B and Zone C at start of planning horizon

$L\,F_1$ and $L\,F_2$ are the loss factors of transmission lines between A and B and B and C respectively.

Additional constraints for limitation of the total installed capacity at the alternative hydro sites

$$\sum_{t=1}^{6} x_{it} \leq 1$$

for i = hydro alternatives

This model has got in total 199 variables and 54 constraints.

9.2 OPERATIONAL PLANNING MODEL FOR THE REGION

To find the minimum annual cost operation of the electricity system as it expands over the planning horizon according to the schedule determined by the capacity expansion model, the hydro planning sub-model, first calculates the operation of hydro-electric plants that maximise their firm on-peak energy. This energy is subtracted from the projected annual energy demands.

The operational planning model then calculates the annual energy generation (and thus the plant factor) of each plant during each period to get a minimum discounted cost of operation over the planning horizon. It is appropriate to emphasise that the purpose of the operational planning sub-problem is not to obtain a policy that will be recommended as the operating procedure to be followed in future since only the expected variable (operating and maintenance costs plus fuel costs) costs per unit of energy are included in the operational planning sub-model.

Having found the system expansion policies from the capacity expansion sub-model and knowing the maximum firm on peak available energy from the hydro-electric plants in each zone, the operational planning model can be formulated.

As observed from the general formulation of the operation planning model, the objective function consists of the sum of total discounted fuel costs plus plant operation and maintenance costs. In particular the net heat rate function, H (number of BTU/KWH) which determine the fuel costs are very difficult to identify without adequate data and more importantly without knowing future plant operating policy. Hence as per the data supplied, we assume 2.5 KWH/ unit of fuel burnt at beginning of policy. This makes the operational planning problem an uncoupled linear program, one for each year of the operational planning horizon.

The variable cost and fixed cost associated with each plants are calculated for a rate of discount of eight percent. This data has been pronded by the planning authorities directly.

The operational planning for the <u>first iteration</u> has been formulated as a series of linear programming problems, one for each period (one year) of the planning horizon.

The general <u>form at year t</u> is given by

$$\text{Minimise} \left\{ \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \left[f_{ijt} + m_{ijt} \right] d_{ijt} \right\}$$

such that

$$d^{b}_{ijt} \geq E^{b}_{jt} \pm \sum_{j} \sum_{i} e^{b}_{ijt}$$

$$d^{p}_{ijt} \geq E^{p}_{jt} \pm \sum_{j} \sum_{i} e^{p}_{ijt} \qquad j = 1 \ldots N_2$$

$$d_{ijt} \leq G_{ijt}$$

$$i = 1, \ldots N_1$$

$$j = 1, \ldots N_2$$

where

f_{ijt} = total fuel cost/unit of energy

$t = 1$ - generates the first year operational planning model

$t = 2$ - generates the second year operational planning model and so on.

f_{ijt} are calculated from the equation

$$f_{ijt} = \frac{\text{fuel costs (= cost per unit amount)}}{\text{(KWH produced per unit amount)}}$$

The operational planning studies for year 1971 has a constraint matrix of dimension 15 x 9. A small programme was written to generate the cost matrices. The L.P. package programme available at the Computer Centre, I.I.T. Kanpur has been used for solving this operational planning problem.

9.3 FORMULATION OF THE CAPACITY EXPANSION MODEL TO BE USED FOR COMPUTATION

From the data available it is possible to derive in a straight forward manner the integer programming model for the system.

The objective function is formulated as

$$\text{Minimise} \left\{ \sum_{i=1}^{97} C_i \cdot x_i + \sum_{j=1}^{24} k_j \quad Z_i \right\}$$

where,

x_i, and Z_i are 0 - 1 variables.

C_i's and k_j's are the discounted total costs of installation, operation and maintenance of plants and transmission lines respectively.

The satisfaction of base and peak demand constraints for Zone A at every period is given bye the following constraints imposed on the objective function:

$$100\ X_1 \times 25\ X_2 + 25\ Y_4 + 50\ Y_5 + 100\ Y_6 \geqslant 1 \text{ (Base demand)}$$

$$45\ X_3 \times 15\ X_4 + 5\ Y_1 + 10\ Y_2 + 15\ Y_3 \geqslant 12 \text{ (Peak demand)}$$

.
.
.

$$100\ X_1 + 25\ X_2 + 25\ X_5 + 100\ X_6 + 25\ X_{10} + 50\ X_{14} + 150\ X_{15}$$
$$+ 150\ X_{26} + 300\ X_{27} + 100\ X_{84} + 25\ Y_{41} + 50\ Y_{42}$$
$$+ 100\ Y_{43} + 150\ Y_{44} \geqslant 629 \quad \text{(Base)}$$

$$30\ X_3 + 60\ X_4 + 30\ X_7 + 60\ X_8 + 90\ X_9 + 60\ X_{12} + 120\ X_{13}$$
$$+ 11\ X_{16} + 7\ X_{17} + 60\ X_{18} + 120\ X_{19} + 11\ X_{22} + 7\ X_{23}$$
$$+ 90\ X_{24} + 120\ X_{25} + 7\ X_{29} + 180\ X_{82} + 270\ X_{82} + 11\ X_{86}$$
$$+ 7\ X_{87} + 30\ Y_{37} + 60\ Y_{38} + 90\ Y_{39} + 120\ Y_{40} \geqslant 640$$

(Peak)

Similar equations can be formulated for other zones.

9.4 DISCUSSION OF NUMERICAL RESULTS OBTAINED FOR THE SELECTED CASE STUDY

The alternatives chosen for plants, transmission lines, and exportation - importation by the capacity expansion model at first iteration are shown in Tables 24,25 & 26.

The results for the first iteration of the operational planning model is shown in Figures 9.1 to 9.3. The thirty year estimated and calculated plant factors for chosen plants are given in Table 27.

The first observation that can be made from Fig. 9.1 is that d_1 (energy from the first unit added to the system) is zero until the end of 1971 - 75. Similarly d_4 (energy from first unit added to the Obregon region, and fourth in sequence of installation) is not needed until the second year of period 1976 - 80, and $d_2 = 0$ till the beginning of 1976 - 80 period.

TABLE 24

Sequences of installations of power plants.
(Results from capacity expansion sub-problem)

ZONE	Project chosen (or existing) 1971	Initial variable O & M and fuel costs. Mills/KWH	Decision variables	Year of installation
1	2	3	4	5
A	Guyamas 96MW(S)	4.2	e_1	-
B	Obregon 28MW(G)	10.4	e_2	-
C	Mochis 41MW(S)	4.1	e_3	-
C	Culiacan 14MW(G)	10.3	e_4	-
A	Guyanas 100MW(S)	4.2	d_1	1971
A	Hernosille 15MW(G)	10.6	d_2	1971
B	Novajoa 50MW(S)	4.4	d_3	1971
B	Obregon 90MW(G)	10.5	d_4	1971
C	Mochis 100MW(S)	4.1	d_5	1971
A	Hernosille 90MW(G)	11.1	d_6	1976
B	Novajoa 50MW(S)	4.4	d_7	1976
B	Obregon 60MW(G)	10.5	d_8	1976
C	Mochis 100MW(S)	4.1	d_9	1976
C	Huites 100MW(H)	-	-	1976
A	Caborca 50MW(S)	5.0	d_{10}	1981
A	Hernosille 60MW(G)	11.1	d_{11}	1981
B	Novajoa 50MW(S)	4.5	d_{12}	1981
B	Obregon 120MW(G)	10.7	d_{13}	1981
C	Mochis 100MW(S)	4.1	d_{14}	1981
C	Culiacan 60MW(G)	10.5	d_{15}	1981
C	Culiacan 120MW(G)	10.6	d_{16}	1981
C	Lopezmateos 790MW(H)	-	-	1981
A	Caberca 75MW(S)	5.0	d_{17}	1986
A	Hernosille 120MW(G)	11.2	d_{18}	1986
B	Novajoa 50MW(S)	4.1	d_{19}	1986

(Table contimued)

TABLE 24 (Continued)

1	2	3	4	5
B	Novajoa 100MW(S)	4.5	d_{20}	1986
C	Mochis 500MW(S)	5.2	d_{21}	1986
C	Culiacan 400MW(G)	10.7	d_{22}	1986
A	Caborca 300MW(S)	5.4	d_{23}	1991
A	Hernosille 180MW(G)	11.2	d_{24}	1991
B	Novajoa 150MW(S)	4.5	d_{25}	1991
B	Obregon 120MW(G)	10.6	d_{26}	1991
C	Mochis 500MW(S)	5.2	d_{27}	1991
C	Culiacan 400MW(G)	10.7	d_{28}	1991
A	Caborca 300MW(S)	5.4	d_{29}	1996
A	Hernosille 240MW(G)	11.2	d_{30}	1996
B	Novajoa 300MW(S)	4.5	d_{31}	1996
B	Obregon 50MW(G)	10.5	d_{32}	1996
B	Obregon 120MW(G)	10.6	d_{33}	1996
C	Mochis 200MW(S)	4.5	d_{34}	1996
C	Culiacan 400MW(G)	10.7	d_{35}	1996

TABLE 25

Import - Export Alternative
Chosen at second iteration

Import to Zone	Type of capacity	Amount M.W.	Period during which transfer occurs
A	Base	25	71 - 75
A	Peak	15	71 - 75
A	Base	50	96 - 2000
A	Peak	90	96 - 2000
B	Base	45	71 - 75
B	Peak	60	71 - 75
B	Base	120	71 - 75
B	Peak	90	96 - 2000
B	Peak	60	76 - 80

TABLE 26

Transmission line alternatives
Chosen at second iteration

Zones connected	Type	Maximum transmission capacity (MW)	Year to start
B - A	220 KV S.C.R	300	1995
C - B	220 KV S.C.R	300	1971

TABLE 27

First iteration operation planning sub-problem results

Decision variables	Estimated average plant factor %	Calculated average plant factor %	Decision variable	Estimated average plant factor %	Calculated average plant factor %
e_1	55	82	d_{17}	55	66
e_2	15	23	d_{18}	15	24
e_3	55	85	d_{19}	55	85
e_4	15	22	d_{20}	55	64
d_1	55	61	d_{21}	55	68
d_2	15	15	d_{22}	15	22
d_3	55	81	d_{23}	55	64
d_4	15	9	d_{24}	15	17
d_5	55	72	d_{25}	15	12
d_6	15	17	d_{26}	15	20
d_7	55	73	d_{27}	15	36
d_8	15	8	d_{28}	15	26
d_9	55	63	d_{29}	55	16
d_{10}	55	60	d_{30}	15	27
d_{11}	15	25	d_{31}	55	68
d_{12}	55	22	d_{32}	15	27
d_{13}	15	19	d_{33}	15	7
d_{14}	55	63	d_{34}	55	85
d_{15}	15	28	d_{35}	15	18
d_{16}	15	26			

Since this represents extra unused capacity, in the next capacity expansion problem, we can delay these projects until 1976 - 80, time, period. Using the same reasoning construction of the plant producing d_8 can be delayed until 1981 - 85, and construction of plants producing d_{14} and d_{16} until 1986 - 90. From Figures 9.1 to 9.3 we get information regarding the time period of construction and can utilise them for the next iteration of the capacity expansion prlblem. Similar interpretation can be given for other values.

The second iteration can now proceed for the capacity expansion problem with values of plant factors obtained from the first iteration of the operational planning problem. This iterative process can be continued till no further improvement in capacity installation sequences occur or the sequence converges. The total cost of installation at the first iteration with assumed values of parameters turns out to be 4896 million dollars. The second iterational capacity expansion total cost is to the tune of 5225 million dollars. The second iteration uses different plant factor values (those obtained from first iteration operational planning model) and hence this difference in total costs are obtained.

9.5 SUMMARY AND CONCLUSIONS

The purpose of this study was to construct a mathematical model for determining the best locations, time, and size of future power plants in a region. The economic objective was the minimisation of total discounted cost, given a projected demand for electricity.

The problem of planning the best capacity expansion of an electrical system has been treated in the literature using qualitative approaches, plant by plant analysis, simulation and optimisation techniques, applied strictly to strategic (capacity expansion) aspect of the problem. Variable operating costs show to be a significant part of the total costs. Hence careful analysis of the operational aspects of the problem is essential, if a truely minimum total cost capacity expansion policy is desired. A methodology for tackling both the capacity expansion problem and operational planning problem in an iterative way using adaptive modelling ideas at each iteration has been proposed. Considerable improvements in costs can be obtained by using the iterative methodology.

The problem of dimensionality limits the size of the problem that can be solved by this methodology. The integer code for capacity expansion sub-problem is based on Bala's algorithm (201). Due to the time cost limitations, a trade off between optimality of solution

and computation costs are necessary. However, the operational planning L.P. problem is a systematic search procedure which guarantees optimal solution.

Models are simplifications of reality and the one presented in this study is no exception. However, the main aspects of the electrical power system capacity expansion problem are included. Assumptions such as the deterministic character of the model were made with the ultimate goal of compromising optimality of solution with realism of the problem that can be solved. As more efficient algorithms are developed, better and more realistic problems can be solved with the methodology presented in this dissertation.

Planning studies such as the one presented here are usually complicated by the need to accurately project possible future economic conditions of the area. This is a difficult task (if not impossible to do) because of the many unexpected events that usually occur in a region. This problem can however be circumvented by a repeated solution of the problem $T > \Delta T$ years (such as five years) and implement the decision for the first ΔT years of the expansion policy obtained each time. This adaptive modelling process therefore makes use of both a planning horizon of length T and an implementation period of $\Delta T < T$.

9.6 SCOPE FOR FURTHER STUDY

The following topics are recommended as fruitful areas for further study:

1. Testing the methodology presented in this dissertation using the mixed integer capacity expansion model, where the continuous variables would represent the export, import decisions. This requires the development/or implementation of an efficient algorithm capable of solving realistic problems.

2. Further investigation and numerical comparison of Bender's decomposition approach as modified by Geoffrion (284) with the methodology suggested in this dissertation for larger problems appear specially fruitful.

3. Development of stochastic decision making models for further investigation of the reliability aspects of the capacity expansion problem.

4. Investigation of the dual of the operational planning problem and its role in the adaptive procedure used in this dissertation.

5. Development of models and systematic procedures or algorithms that combined with experience could be used for screening out alternatives. This area is of special importance as the methodology of this dissertation assumes that a good screening study has been made before the capacity expansion problem is solved.

6. Development of models for finding the optimum conjuctive operation of the hydroelectric and other plants in each zone.

7. Development of sub-models to the capacity expansion problem that might help the planners to systematically quantify some of the intangible aspects of the problems like pollution and environment.

REFERENCES

1. Report of the Energy Survey of India Committee, Government of India, New Delhi, 1965.

2. Davar, G.R., "The Energy Crisis and its Impact on Planning and Administration", Indian Journal of Public Administration, Vol. 21, No. 4, October - December, 1975, p. 762.

3. Brown, R.G., Statistical Forecasting For Inventory Control, McGraw Hill Pub. Co., New York, 1950.

4. Brown, R.G., Smoothing, Forecasting, and Predictions of Time Series, Prentice Hall, London, 1963.

5. Brown, R.G., Exponential Smoothing For Predicting Demand, ORSA, Nov. 16, 1956.

6. Brown, R.G., Decision Rules For Inventory Management, Holt, Rinehart and Winston, New York, 1957.

7. Brown, R.G., Economical Safety Stocks, ORSA, May 10, 1957.

8. Brown, R.G. and R.F. Meyer., "The Fundamental Theorem of Exponential Smoothing," Journal of the Operations Research Society of America, Vol. 9, 1961, p. 673.

9. Winters, P.R., "Forecasting Sales by Exponentially Weighted Moving Averages", Management Science, Vol. 6, 1960, p. 324.

10. Winters, P.R., Chapter 14 of Planning Production, Inventory and Work Force, by Holt, Muth, Modigliani et. al., Prentice Hall, New York, 1960.

11. Muth, J.F., "Optimal Properties of Exponentially Weighted Forecasts of Time-Series with Permanent and Transitory Components", Journal of the American Statistical Association, Vol. 55, 1960, p. 299.

12. Geoffrion, A.M., "A Summary of Exponential Smoothing Techniques", Journal of Industrial Engineering, Vol. 13, No. 4, July-Aug. 1962, p. 223.

13. Pegels, C.C., "Exponential Smoothing - Some New Variations", Management Science, Vol. 15, No. 4, July - Aug. 1962, p. 223.

14. Wiener, H., "Extrapolation, Interpolation and Smoothing of Stationary Time-Series" John Wiley Pub. Co., New York, 1949.

15. Duffin, R.J. and P. Whiddin, "An Exponential Extrapolator", Technical Report No. 46, Dept. of Mathematics, Carnegie Institute of Technology, U.S.A., 1960.

16. Duffin, R.J. and T.W. Schmidt, "An Extrapolator and Scrutator", Journal of Math. Analysis and Applications, Sept. 1960.

17. Harrison, P.J., "Short Term Sales Forecasting", Applied Statistics, Vol. 16, No. 2 - 3, 1965, p. 314.

18. Harrison, P.J., "Exponential Smoothing and Short Term Sales Forecasting", Management Science, Vol. 13, No. 11, July 1969, p. 821.

19. Harrison P.J., and O.L. Davies, "The Use of Cusum Techniques for Control of Routine Forecasts of Product Demand", Operations Research, Vol. 12, No. 2, 1964, p. 127.

20. Harrison P.J. and F.A. Scott, "A Development of a System for Use in Short Term Sales Forecasting Investigation", Paper presented at the O.R.S.A. Annual Conference, Shrivenham, Nov. 1965.

21. Kirby, R.M., "A Comparison of Short and Medium Range Statistical Forecasting Methods", Management Science, Vol. 13, No. 4, Dec. 1966., p. B-202.

22. Shiskin, Julius., "Seasonal Adjustment by Electronic Computer Method", Technical Report No. 12, National Bureau of Economic Research, U.S.A, 1958.

23. Holt, C.C., "Forecasting Seasonal and Trend by Exponentially Weighted Moving Averages", Carnegie Institute of Technology, Pittsburgh, Pensyllvania, U.S.A., 1957.

24. Holt, C.C., Chapter 14 of Planning Production, Inventory and Work Force, by Holt, Modigliani, Muth and Simon, Prentice Hall, London, 1960.

25. Cox, D.R., "Prediction by Exponentially Weighted Moving Average and Related Methods", Journal of the Royal Statistical Society, Series B, Vol. 23, 1961, p. 414.

26. Buffa, E.S., Modern Production Management, Richard D. Irwin Inc., Homewood, Illinois, 1969.

27. Buffa, E.S. and W.H. Taubert, Production - Inventory System, Planning and Control, Richard D. Irwin, Inc., Homewood, Illinois, 1972.

28. Arrow, K.J., S. Karlin, and H. Scarf, Studies in the Mathematical Theory of Inventory Control Stanford University Press, Stanford, 1958.

29. Whitin, T.A., Theory of Inventory Management, Princeton University Press, N.J., 1953.

30. Welch, W.F., Scientific Inventory Control, Management Pub. Co., Greenwich, Conn., 1956.

31. Magee, J.F. and D.M. Boodman, Production Planning and Inventory Control, McGraw Hill Book Co., New York, 1967.

32. Moore, F.G. and R. Jablonski, Production Control, McGraw Hill Book Co., New York, 1969.

33. Greene, J.H., Production Control Systems and Decisions, Richard D. Irwin Co., Homewood, Illinois, 1965.

34. Eilon, S., Elements of Production Planning and Control, Mcmillan Co., New York, 1962.

35. Buchan, J. and E. Koenigsberg, Scientific Inventory Management, Prentice Hall, Englewood Cliffs, N.J., 1963.

36. Theil, H. and S. Wage, "Some Observations on Adaptive Forecasting", Management Science, Vol. 10, No. 2, Jan. 1964, p. 762.

37. Nerlove, M. and S. Wage, "On the Optimality of Adaptive Forecasting" Management Science, Vol. 10, No. 2, Jan. 1964, p. 207.

38. Chow, W.M., "Adaptive Control of Exponential Smoothing Constants", Journal of Industrial Engineering, Vol. 3, No. 6, June 1971, p. 15.

39. Dudman, R.S., "Forecasting with Three Parameter Adaptive Smoothing", 13th International Meeting of TIMS, Philadelphia, Sept. 1966.

40. Wheelright S.C. and Mabridabis, S., "An Examination of The Use of Adaptive Filtering in Forecasting", O.R. Quarterly, Winter 1973, p. 326,

41. Jain, H.K. "An Adaptive Approach For Forecasting and Inventory Control, M.Tech. Thesis, Deptt. of Mech. Engg., I.I.T. Kanpur, Feb. 1974.

42. Packer, A.H.,"Simulation and Adaptive Forecasting as Applied to Inventory Control", Operations Research, Vol. 15, No. 4, July-Aug., 1967, p. 600.

43. Trigg, D.W. and A.G. Leach - "Exponential Smoothing With An Adaptive Response Rate", Operation Research Quarterly, Vol. 18, No. 1, 1967.

44. Box, G.E.P. and G.M. Jenkins,"Some Statistical Aspects of Adaptive Optimisation and Control", Journal Royal Stat. Soc., Sec. B., Vol. 24, 1962, p. 297.

45. Box, G.E.P. and G.M. Jenkins, Time Series Analysis Forecasting and Control, Holden-Day, San Francisco, Cambridge, London, 1970.

46. McClain, J.O. and L.J. Thomas,"Response Variance Trade Offs in Adaptive Forecasting", Operations Research, Vol. 21, No. 2, March-April, 1973, p. 554.

47. McClain, J.O.,"Dynamics of Exponential Smoothing with Trend and Seasonal Term", Management Science, Vol. 20, No. 9, May 1974.

48. Morris, R.H. and C.R. Glassey, "The Dynamics and Statics of Exponential Smoothing Operations", Operations Research, Vol. 11, July-Aug, 1963, p. 561.

49. Buffa, E.S., Operations Management, Wiley Pub. Co., New York, 1972.

50. Griffin, W.C., Introduction to Operations Engineering Irwin Pub. Co., Homewood, Illinois, 1971.

51. New, W.R., "Load Forecasting on The TVA System" AIEE Trans., PAS, Vol. 81, June 1962, p. 101.

52. Doobie, J.M., "Forecasting Periodic Trends by Exponential Smoothing", Operations Research, 1963, p. 908.

53. Christianse, W.R., "Short Term Load Forecasting Using General Exponential Smoothing", IEEE Trans., PAS, Vol. 90, No. 2, March-April, 1971.

54. Gupta P.C. and K. Yamada, "Adaptive Short Term Forecasting of Hourly Loads Using Weather Information", IEEE Trans, PAS, Vol. 91, No. 5, Sept.-Oct, 1972.

55. Berry, T.W. and I.J. Whitting, "Alternative Plans For An Expanding Electric Power System", Queen Marry College Research Report, London Univ., Sept. 16-19, 1963.

56. Cowden, D.J., "The Perils of Polynomial", Journal of Institute of Mathematical Sciences, Vol. 9, July 1963, p. 542.

57. Heineman, G.T., D.A. Nordman and E.C. Plant, "The Relationship Between Summer and Weather Loads - A Regression Analysis", IEEE Trans.,PAS 85, Nov. 1966, p. 1144.

58. Chen, G.K.C. and P.R. Winters, "Forecasting Peak Demand of An Electricity System With a Hybrid Exponential Model", Management Science, Vol. 12, No. 12, Aug. 1966.

59. Yamada, K, "A General Hybrid Regression Forecasting Model and Its Applications", Mitsubishi Electrical Corporation, Tokyo, Japan.

60. Johnston, J., Econometric Methods, McGraw Hill Book Co., N.Y., 1971.

61. Klein, L.R., A Textbook Of Econometrics, Row Peterson, Evanston, Illinois, 1953.

62. Goldberger, A.S., Econometric Theory, Wiley Publications, N.Y., 1964.

63. Theil, H., Applied Economic Forecasting, North Holland Pub. Co., 1966.

64. Theil, H., Economic Forecasts and Policy, North Holland Pub. Co., Amsterdam, 1961.

65. Theil, H., Principles of Econometrics, North Holland, Amsterdam, 1970.

66. Dhrymes, P.J., Econometrics - Statistical Foundations and Applications, Harper and Row, N.Y. 1970.

67. Christ, C.F., 'Econometric Models and Methods, John Wiley and Sons, N.Y., 1966.

68. Tinbergen, J., Econometrics (Translated by H. Rijkenvan Olst from Dutch) Blackinston, Philadelphia, 1951.

69. Tintner, G., Econometrics, Wiley, N.Y., 1952.

70. Wannacot, R.J., Econometrics, Wiley, N.Y., 1970.

71. Lesser, C.E.V., Econometric Techniques and Problems, Charles Griffin and Co., London, 1966.

72. Cramer, J.S., Empirical Econometrics, North Holland, Amsterdam, 1969.

73. Hood, W.C. and T.C. Koopmans(ed.), Studies in Econometric Method, Cowles Commission, Monograph, No. 14, Wiley, N.Y., 1953.

74. Koopmans, T.C., Linear Regression Analysis of Economic Time Series, Haarlem, De Erven F. Bohn, N.V., 1937.

75. Wold, H.A., Econometric Model Building Essays on the Causal Chain Approach, North Holland Pub. Co., Amsterdam, 1964.

76. Buttler, W.F., R.A. Kavesh and R.B. Platt, Methods and Technique of Business Forecasting, Prentice Hall, N.J., 1974.

77. Chisholm, R.K. and G.W. Whitaker, Forecasting Methods Richard D. Irwin & Co., Homewood, Illinois, 1971.

78. Maulinvaud, E., Statistical Methods in Econometrics North Holland Publishing Co., 1966.

79. Suits, D.B.,"Forecasting and Analysis with an Econometric Model", The American Economic Review. Vol. 52, 1962, p. 104.

80. Stekler, H.O., "Forecasting with Econometric Models an Evaluation", Econometrica, Vol.36, 1968, p. 437.

81. Leontief, W., Input Output Economics, Oxford University Press, N.Y., 1966.

82. Leontief, W., The Structure of the American Economy, Harvard University Press, Massachussets, Cambridge, Mass. 1951.

83. Leontief, W., Studies in the Structure of the American Economy, Oxford University Press, New York, 1953.

84. Mogenstern, O., Economic Activity Analysis, John Wiley and Sons, New York, 1954.

85. Rasmussen, P.N., Studies in the Intersectoral Relationships, North Holland Publishing Co., Amsterdam, 1957.

86. Stone, R.A., Input Output and National Accounts, OEEC, Paris, 1961.

87. Dorfman, R., "The Nature and Significance of Input Output", Review of Economic Statistics, Vol. 55, 1954, p. 121.

88. Dorfman, R., P.A. Samuelson, R.A. Solow, Linear Programming and Economic Analysis, McGraw Hill Pub. Co., London, 1958.

89. Koopman, T.C., Activity Analysis of Production and Allocation, John Wiley and Aons, New York, 1951

90. Chenery, H.B. and P.G. Clark, Inter-industry Economics John Wiley & Sons, N.Y., 1965.

91. Mathur, P.N. and R. Bharadwaj, Economic Analysis in Input Output Framework With Indian Empirical Explorations, Input Output Research Association, Poona, India, 1967.

92.. Wold, H.A. and L. Jureen, Demand Analysis A Study in Econometrics - Wiley, N.Y., 1953.

93. Taylor, L.D., "The Demand for Electricity : a Survey" Bell Journal of Economics, Spring 1974, p. 117.

94. Parks, R.W., "Systems of Demand Functions: An Empirical Comparison of Alternative Functional Forms", Econometrica, Vol. 37, Oct. 1969, pp. 77-96.

95. Houthakker, H.S. and L.D. Taylor, Consumer Demand in The United States, 2nd ed., Harvard Univ. Press, Cambridge, 1970.

96. Philips, L., "A Dynamic Version of the Linear Expenditure Model", Review of Economics and Statistics, Vol. 54, No. 4, Nov. 1972, p. 445.

97. Taylor, L.D. and D. Weiserbs, "On the Estimation of Dynamic Demand Functions", Review of Economics and Statistics, Vol. 54, No. 4, Nov. 1972, p. 459.

98. Brown, M. and D. Heien, "The S-Branch Utility Tree: A Generalization of the Linear Expenditure System", Econometrica, Vol. 40, July 1972, p. 737.

99. Christensen, L.T. et. al., "Transcendental Logarithmic Utility Functions", Discussion Paper No. 285, Harvard Institute of Economic Research, March 1973.

100. Houthakker, H.S., "Some Calculations of Electricity Consumption in Great Britain", Journal of the Royal Statistical Society (A), Vol. 114, Part III, 1951, p. 351.

101. Buchanan, J.M., "Peak Loads and Efficient Pricing: Comment", Quarterly Journal of Economics, August 1966, p. 463.

102. Gabor, A., "Further Comment (On Peak Loads and Efficient Pricing)", Quarterly Journal of Economics, Vol. 80, August 1966, p. 472.

103. Gabor, A., "A Note on Block Tariffs", Review of Economic Studies, Vol. 23, 1955-56, p. 32.

104. Oi, W.Y., "A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly", Quarterly Journal of Economics, Vol. 85, Feb. 1971, p. 77.

105. Houthaker, H.S., "Some Calculations of Electricity Consumption in Great Britain", Journal of the Royal Statistical Society (A), Vol. 114, Part III, (1951), p. 351.

106. Fisher, F.M. and C. Kaysen, A Study in Econometrics : The Demand for Electricity in the United States, North Holland Publishing Co., 1962.

107. Baxter, R.E. and Rees, R., "Analysis of the Industrial Demand for Electricity", American Economic Review, Vol. 53, 1963, p. 207.

108. Wilson, J.W., "Residential Demand for Electricity", Quarterly Review of Economics and Business, Vol. 11, No. 1, 1971, p. 7.

109. Boiteux, M., "Electrical Energy: Facts, Problems and Prospects," in J.R. Nelson, ed., Marginal Cost Pricing in Practice, Prentice Hall, 1964.

109a. Houthakker, H.S., "Electricity Tariffs in Theory and Practice", Electricity in the United States, Amsterdam: North Hollan Publishing Co., 1962.

110. Lewis, W.A. "The Two Part Tariff", Economica, Vol. 8, 1941, p. 249.

111. Cargil, T.E. and R.A. Meyer, "Estimating the Demand For Electricity", Applied Economics, Vol. 3, 1971, p. 233.

112. Anderson, K.P., "Toward Econometric Estimation of Industrial Energy Demand: An Experimental Application to the Primary Metals Industry," The Rand Corporation (R-719-NSF), December 1971.

113. Mount, T.D., Chapman, et. al., "Electricity Demand in the United States: An Econometric Analysis," Oak Ridge National Laboratory, Oak Ridge, Tenn., June 1973.

114. Anderson, K.P., "Residential Demand For Electricity: Econometric Estimates for California and the United States," The Rand Corporation, January 1972.

115. Lyman, R.A., "Price Elasticities in the Electric Power Industry," Department of Economics, University of Arizona, October, 1973.

116. Box, G.E.P. and D.R. Cox., "An Analysis of Transformations", Journal of the Royal Statistical Society, Series B., Vol. 26, No. 2, 1964, p. 211.

117. Zarembka, P., "Functional Form in the Demand For Money", Journal of the American Statistical Association, Vol. 63, No. 2, June 1968, p. 502.

118. Zellner, A., An Introduction to Bayesian Inference in Econometrics, New York, John Wiley & Sons, N.Y., 1971.

119. Houthakker, H.S., Veerleger, P.K. and Sheehan, D.P., "Dynamic Demand Analysis for Gasoline and Residential Electricity"., American Journal of Agriculture Economics, Vl. 56, No. 2, May, 1974, p. 412.

120. Balestra, P. and M. Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of A Dynamic Demand Model", Econometrica, Vol. 34, July 1966, p. 585.

121. Nerlove, M., "A Note on Error Component Models", Econometrica, Vol. 39, March, 1971, p. 383.

122. Cicchetti, C.J. and V.K. Smith, "Alternative Price Measures and the Residential Demand for Electricity", Regional Science and Urban Economics, Vol. 5, No. 4, Dec. 1975, p. 503.

123. Ramsey, J.B., "Models, Specification Errors and Inference - A discussion of Some Problem in Econometric Methodology", Bulletin of the Oxford Institute of Economics and Statistics, Vol. 32, Nov. 1969.

124. Ramsey, J.B., "Tests for Specification Errors in Classical Linear Least Squares Regression Analysis", Journal of the Royal Statistical Society, 1969, B-31.

125. Ramsey, J.B., "Classical Model Selection Through Specification Error Tests - In Frontiers in Econometrics, P. Zarembka, ed., Academic Press, 1974.

126. Ashbury, J.G., "The Econometric Approach to Energy Supply and Demand", Review and Analysis, Argonne National Laboratory, May 1974.

127. Halvoresen, R., "Residential Demand For Electricity", The Review of Economics and Statistics, Vol. 57, No. 1, Feb. 1975, p. 12.

128. Halvoresen, R., "Residential Electricity Demand and Supply," Presented at the Sierra Club Conference on Power and Public Policy, Vermont, January 1972.

129. Halvoresen, R., "Long Run Residential Demand For Electricity", Discussion Paper No. 73-6, Institute of Economic Research, University of Washington, U.S.A., 1973.

130. Halvorsen, R., "Demand for Electricity Power in the United States", Discussion Paper No. 73-13, Institute of Economic Research, University of Washington, 1973.

131. Hawkins, R.G., "The Demand For Electricity A Cross Section Study of New South Wales Territory", The Economic Record, Vol. 51, No. 133, March 1975, p. 1.

132. Wilson, J.W., "Electricity Consumption Supply Requirements, Demand Elasticity and Rate Design", American Journal of Agricultural Economics, Vol. 56, No. 2, May, 1974, p. 419.

133. Tyrell, T.J., "Projections of Electricity Demand", Oak Ridge National Laboratory, Oak Ridge, Tennesse, Nov. 1973.

134. Chapman, D., Mount, et. al., "Electricity Demand Growth and The Energy Crisis", Science, Vol. 178, Nov. 1972, p. 703.

135. Hannan, E.J., Time Series Analysis, Methuen, London, 1960.

136. Wold, H., A Study in The Analysis of Stationary Time Series, Alnquist and Wiksell, Stockholm, 1954.

137. Wold, H., Bibliography on Time Series and Stochastic Processes, Oliver and Boyd Ltd., Edinburgh, 1965.

138. Whittle, P., Prediction and Regulation by Linear Least Square Methods, English University Press, London, 1963.

139. Box, G.E.P., G.M. Jenkins and D.W. Bacon, "Models for Forecasting Seasonal and Non-Seasonal Time Series", Advanced Seminar on Spectral Analysis of Time Series, ed. B. Harris, 271, John Wiley, New York, 1967.

140. Box, G.E.P. and G.M. Jenkins, "Some Recent Advances in Forecasting and Control, I," Applied Stat., Vol. 17, No. 91, 1968.

141. Box, G.E.P. and G.M. Jenkins, "Discrete Models for Forecasting and Control", Encyclopedia of Linguistics, Information and Control, 162, Pergamon Press, 1969.

142. Box, G.E.P. and G.M. Jenkins, "Some Statistical Aspects of Adaptive Optimization and Control", Journal of Royal Statistical Society, Vol. B24, No. 297, 1962.

143. Bartlett, M.S., An Introduction to Stochastic Processes, Cambridge University Press, London, 1955.

145. Doob, J.L., Stochastic Processes, John Wiley, New York, 1953.

146. Parzen, E., Stochastic Processes, Holden Day, San Francisco, 1962.

147. Anderson, T.W., The Statistical Analysis of Time Series, John Wiley & Sons., Inc., New York, 1971.

148. Ivankhenko, A.G. and V.G. Lapa, Cybernetics and Forecasting Techniques, American Elseviere Publishing Co., New York, 1967.

149. Quenouille, M.H., Analysis of Multiple Time Series, Hafner, New York, 1957.

150. Rosenblatt, M. and U. Grenander, Statistical Analysis of Stationary Time Series, Wiley, New York, 1957.

152. Kolmogorov, A.N., "Interpolation and Extrapolation of Stationary Random Sequences," Bulletin of the Moscow State University, Vol. 2, No. 6, 1941

153. Pugachev, U.S., Theory of Random Functions and Its Application to Control Problems, Addison Wesley, N.Y., 1963.

154. Borde, H. and C.E. Shannon, "A Simplified Derivation of the Linear Least Square Smoothing and Prediction Theory", Proc. IRE, Vol. 38, No. 4, April 1950, p. 417.

155. Gabor, D., W. Wilby et. al., "A Universal Non-Linear Filter Predictor and Simulator Which Optimises Itself by a Learning Process", Proc. IEE, Vol. 108B, No. 40, 1961, p. 422.

156. Lubceck, J.K., "The Optimisation of a Class of Non-Linear Filters", Proc. IEE, Vol. 107C, Nov. 1969, p. 69.

157. Crammer, S and H. Leadbetter, <u>Stationary and Related Stochastic Processes</u>, John Wiley, N.Y., 1958

158. Cox, B.R., and G.H. Miller, <u>A Theory of Stochastic Processes</u>, John Wiley, N.Y., 1958.

159. Zadeh, L.A. and J.R. Ragazzini, "An Extension of Wieners Theory of Prediction, Journal of App. Physics., Vol. 21, No. 7, 1950, p. 645.

160. Prestley, M.B., "Evolutionary Spectra and Non-Stationary Process," Paper Read at Research Methods Meeting of the Society, University of Manchester Feb. 3, 1965.

161. Rao, A.G. and Shapiro, A., "Adaptive Smoothing Using Evolutionary Spectra," Management Science, Vol. 17, No. 3, Nov. 1970, p. 208.

162. Couts, D. D. Grether, et. al., "Forecasting Non-Stationary Economic Time Series," Management Science, Vol. 13, No. 1, Sept. 1966.

163. Farmer, E.D., "A Method of Prediction for Non-Stationary Processes and Its Application to the Problem of Load Estimation", Proceedings of the Second Congress of I.FAC, Basle, 1963.

164. New, W.R., "Load Forecasting on the TVA System, Part IV - System Forecasting", Presented at IEEE Summer Meeting, Toronto, Canada, June 16-21, 1963.

165. Hammersely, D.G. and H.S. Handscombe, <u>Monte Carlo Methods</u>, John Wiley, New York, 1964.

166. Lathan, N.P. et. al. "A Probabilistic Approach to Electric Utility Load Forecasting", IEEE Summer Power Meeting, Neworleans, July 1966.

167. Stanton, K.N. and P.C. Gupta, "Long Range Load Forecasting of Electricity Utility Systems", Purdue University, Research Report, No. 31, Aug., 1968.

168. Stanton, K.N., P.C. Gupta et. al., "Long Range Demand Forecasting in the Electric Utility Industry", PICA Conference, May 1969.

169. Stanton, K.N. and P.C. Gupta, "Long Range Forecasting and Electricity Demand by Probabilistic Methods", American Power Conference, Chicago, April 24, 1969.

170. Stanton, K.N. and P.C. Gupta, "Forecasting Annual or Seasonal Peak Demand in Electricity Utility Systems", IEEE Summer Power Meeting, Dallas, Texas, June 1969.

171. Gupta, P.C., "Statistical and Stochastic Techniques of Peak Power Demand Forecasting", Purdue Research and Education Centre, Report No. 51, 1969.

172. Toyoda, J and M. Chen et. al., "An Application of State Estimation to Short Term Load Forecasting, Part I - Forecast Modelling", IEEE Trans., Vol. PAS 89, No. 7, Sept.-Oct., 1970, p. 1678.

173. Toyoda J and Y. Inoue, "An application of Sequential Filter to Power System and Separation Algorithms for System Noise and Observation Noise," Presented at IEE (Japan) Sprint Meeting, Tokyo Japan, 1969.

174. Toyoda, J., "Study of Short Range Forecasting Models for Systems Load,", J. IEE (Japan), Vol. 82, No. 883, April 1962, p. 617.

175. Toyoda, J., M. Chen, et. al., "An Application of State Estimation to Short Term Load Forecasting Part II - Implementation", IEEE Trans., Vol. PAS 89, No. 7, Sept. - Oct, 1970, p. 1683.

176. Toyoda J. and Y. Inoue, "An Adaptive Predictor of River Flow For On Line Control of Water Resource Systems," Automatica, Vol. 5, No. 2, March 1969, p. 175.

177. Toyoda, J., "Forecasting Short Term Systems Load", J. IEE (Japan), Vol. 83, No. 889, July 1963, p. 296.

178. Schweeppe, F.C. and J. Weldes, "Power Systems Static State Estimation Pt-I Exact Model", IEEE Trans., Vol. PAS 89, Jan. 1970, p. 120.

179. Larson, R.E and W.F. Tinne et. al., "State Estimation in Power Systems Pt. I Theory and Feasibility", IEEE Trans, Vol. PAS-89, March 1970, p. 345.

180. Kalman R.E. and R.S. Bucy, "A New Approach to Linear Filtering and Prediction Problems", Trans. ASME, J. Basic Engg., Ser. D., Vol. 82, 1960, p. 35.

181. Sorenson, H.W., Kalman Filtering Techniques, Advance Control Systems, 1966.

182. Aoki, M., Optimisation of Stochastic Systems, Academic Press, N.Y., 1967.

183. Sage, A.P. and G.W. Hua, "Algorithms for Sequential Adaptive Estimation of Prior Statistics", Presented at IEEE Symp. on Adaptive Processes, State College, Pa., Nov. 17, 1969.

184. Mehra, R.K., "On The Identification of Variances and Adaptive Kalman Filtering", Proc. JACC, Boulder, Colorado, 1969, p. 494.

185. Mehra, R.K., "On Line Identification of Linear Dynamic Systems with Applications to Kalman Filtering", Proceedings, JACC, Atlanata, 1970.

186. Hookes, R.G., "Forecasting the Demand for Electricity", AIEE Trans., Power APP and Systems, Vol. 74, Oct. 1955, p. 903.

187. Goddard, W., "Electric UtilityLoad Forecasting", IEEE Trans., Vol. PAS 74, 1955, p. 1428.

188. Schiller, P., "Operational Research in the Electricity Supply Industry", Proceedings of Inst. Elect. Engineering, Pt. I, Vol. 198, No. 112, July 1951, p. 247.

189. Felice, J.D., "The Methods of Forecasting Long Term Consumption of Electricity," Congress of Unipede, at Baden, Baden, 1961.

190. Dourille, E., "The Model Coefficients Technique for Forecasting The Perspective Demand for Electricity in the Industries", Ann. Mines, Jan. 1962.

202. Clair, S.S. and W.S. Einwetcher, "The Weather and Daily Forecasting of Philadelphia Electricity Systems Loads", Proc. American Power Conference, Vol. 23, 1961, p. 853.

203. Corpening, S.L. and N.D. Reppen, et. al. "Experience with Weather Sensitive Load Models for Short and Long Range Forecasting", IEEE Trans, PAS, Vol. 92, No. 6, Nov.-Dec., 1973.

204. Frisch, R., Statistical Confluence Analysis by Means of Complete Regression Systems, University Economic Institute, Oslo, Norway, 1934.

205. Samuelson, P.A. and Koopman, T.C., et. al, "Report of Evaluative Committee for Econometrica", Econometrica, Vol. 22, No. 2, April 1954, p. 141.

206. Johansen, L., Production Functions, North Holland Pub., 1972.

207. Kyock, L.M., Distributed Lags and Investment Analysis North Holland Pub., 1954.

208. Box, G.E.P. and G.M. Jenkins, "Models For Prediction and Control", Part I, II, III, IV, V, Univ. of Wisconsin, Dept. of Statistics, Report No. 77, 79, 94 and 95.

209. Jenkins, G.M. and D.G. Watts, Spectral Analysis and Its Applications, Holden-Day, San Francisco, 1968.

210. Wals, A., Sequential Analysis, Wiley & Sons, 1958.

211. Robinson, E.A., Multichannel Time Series Analysis, Holden-Day, San Francisco, 1967.

212. Grenander, U. and M. Rosenblatt, Statistical Analysis of Stationary Time Series, John Wiley, New York, 1957.

213. Rao, C.R., Linear Statistical Inference and Its Applications, Wiley, N.Y., 1957.

214. Carlson, R.F., et. al., "Application of Linear Random Models to Four Annual Stream Flow Series, W.R.R., Vol. 6, No. 4, Aug. 1970, p. 1070.

215. Tukey, J.W., "Discussion Emphasizing the Connection Between Analysis of Variance and Spectrum Analysis" Technometrics, Vol. 3, No. 191, 1961.

216. Anderson, T.W., Statistical Analysis Time Series, John Wiley, N.Y., 1971.

217. Nelson, C.R., Applied Time Series Analysis For Managerial Forecasting, Holde-Day, Inc., San Francisco, 1973.

218. Bartlett, M.S., "On the Theoretical Specification of Sampling Properties of Auto-correlated Time Series" Jour. Royal Stat. Soc. B8, 27, 1946.

219. Yule, G.U., "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," Phil. Trans. A226, 267, 1927.

220. Walker, G., "On Periodicity in Series of Related Terms", Proc. Royal Soc., A131, 518, 1931.

221. Fisher, R.A., Statistical Methods and Scientific Inference, Oliver and Boyd, Edinburgh, 1956.

222. Barnard, G.A., "Statistical Inference," Jour. Royal Stat. Soc., B11, 116, 1949.

223. Birnbaum, A., "On the Foundations of Statistical Inference", Jour. Amer. Stat. Assoc., 57, 269, 1962.

224. Whittle, P., Hypothesis Testing in Time Series Analysis, University of Uppsala Publications, 1951.

225. Durbin, J., "The Fitting of Time Series Models," Rev. Int. Inst. Stat., 28, 233, 1960.

226. Barnard, G.A., G.M. Jenkins and C.B. Winsten, "Likelihood Inference and Time Series", Jour. Royal Stat. Soc., A125, 321, 1962.

227. Kendall, M.G. and A. Stuart, The Advanced Theory of Statistics, Vol. 3, Griffin, London, 1966.

228. Mann, H.B. and A. Wald, "On the Statistical Treatment of Linear Stochastic Difference Equations," Econometrica, 11, 173, 1943.

229. Booth, G.W. and T.I. Peterson, "Non-linear Estimation" IBM Share Program, Pa. No. 687 WL NLI, 1958.

230. Marquardt, D.W., "An Algorithm For Least Squares Estimation of Non-linear Parameters", Jour. Soc. Ind. Appl. Math., 11, 431, 1963.

231. Anscombe, F.J. and J.W. Tukey, "The Examination and Analysis of Residuals", Technometrics, 5, 141, 1963.

232. Daniel, C., "Use of Half Normal Plots in Interpreting Factorial Experiments,", Technometrics, 1, 311, 1959.

233. Durbin, J., "Testing for Serial Correlation in Least-squares Regression When Some of the Regressors are Lagged Dependent Variables," Econometrica, 37, 1969.

234. Box, G.E.P. and D.A. Pierce, "Distribution of Residual Autocorrelations in Autoregressive-integrated Moving Average Time Series Models," Jour. Amer. Stat. Assoc. 64, 1970.

235. Draft 5th Five Year Plan, Planning Commission, Government of India, New Delhi, 1974.

236. Annual Survey of Industries, Central Statistical Organisation, Calcutta, India.

237. Bos, H.C., <u>Spatial Dispersion of Economic Activity</u>, Rotterdam University Press, Rotterdam, 1965.

238. Wolfe, P. and Baumol, W.J., "A Warehouse-Location Problem," Operations Research, No. 6, March-April 1958, p. 252.

239. Kuehn, A.A. and Hamburger, M.J., "A Heuristic Program for Locating Warehouses," Management Science, No. 9, July, 1963, p. 643.

240. Cooper, L., "Location - Allocation Problems," Operations Research, No. 11, May-June 1963, p. 331.

241. Cooper, L., "Heuristic Methods for Location-Allocation Problems," SIAM Review, No. 6, January 1964, p. 37.

242. Feldman, E., F.A.Lehrer et. al., "Warehouse Location under Continuous Economies of Scale," Management Science, 12: May, 1966, p. 670.

243. Vietorisz, T, and A.S. Manne, "Chemical Processes, Plant Location and Economies of Scale," in A.S. Manne and H.M. Markowitz (eds.), Studies in Process Analysis, John Wiley and Sons, Inc., New York, 1963, p. 136.

244. Manne, A.S., "Plant Location Under Economies-of-Scale-Decentralization and Computation," Management Science, No. 11, November 1964, p. 213.

245. Efroymson, M.A., and T.L. Ray, "A Branch-Bound Algorithm for Plant Location," Operations Research, 14, May-June 1966, p. 361.

246. Gregory, J.K., "Optimum Plant Size and Location Under Uncertainty," Unpublished Doctoral Dissertation, Stanford University, 1966.

247. Chenery, H.B., "Overcapacity and the Acceleration Principle," Econometrica, Vol. 20, January 1952, p. 1.

248. McDowell, I., "The Economical Planning Period for Engineering Works," Operations Research, Vol. 8, July-August 1960, p. 533.

249. Manne, A.S., "Capacity Expansion and Probabilistic Growth," Econometrica, Vol. 29, October 1961, p. 632.

250. Coleman, J.R., and R. York, "Optimum Plant Design for a Growing Market," Industrial and Engineering Chemistry, Vol. 56, January 1964, p. 28.

251. Veinott, A.F., Jr., and H.M. Wagner, "Optimal Capacity Scheduling I and II," Operations Research, Vol. 10, July-August 1962, p. 518.

252. Whitin, T.M., The Theory of Inventory Management, end ed., Princeton University Press, Princeton, N.J., 1957.

253. Hadley, G., and T.M. Whitin, Analysis of Inventory Systems, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.

254. Manne, A.S., and A.F. Veinott, Jr., "Optimal Plant Size with Arbitrary Increasing Time Paths of Demand," in Investments of Capacity Expansion: Size, Location and Time-Phasing, George Allen and Unwin Ltd., London, 1967.

255. Ghosh, A., Efficiency inLocation and Inter-regional Flows, North Holland Publishing Company, Amsterdam, 1965.

256. Ward, B., "Linear Programming and Soviet Planning," in J.P. Hardt et. al., (eds.), Mathematics and Computers in Soviet Economic Planning, Yale University Press, New Haven, Conn., 1967.

257. Manne, A.S., Investments for Capacity Expansion: Size, Location and Time-Phasing, George Allen and Unwin Ltd., London, 1967.

258. Kendrick, D., Programming Investment in the Process Industries, Technology Press, Cambridge, Mass., 1967.

259. Zangwill, W.I., "A Dynamic Multi-Product Multi-Facility Production and Inventory Model," Technical Report No. 1, Program in Operations Research, Stanford University, Stanford, California, April 1965.

260. Sreedharan, V.P. and H.H. Wein, "A Stochastic Multistage Multiproduct, Investment Model", SIAM Journal of Applied Mathematics, 15: March 1967, p. 347.

261. Erlenkotter, D., "Pre-investment Planning for Capacity Expansion: a Multi-Location Dynamic Model", Doctoral Dissertation, Graduate School of Business, Stanford University, September, 1969.

262. Galloway, C.D., L.L. Garver et. al., "Generation-Transmission Planning and Economic Evaluation," Proceedings of the Third Power Systems Computation Conference, Rome, 1969.

263. Jacoby, H.D., Analysis of Investment in Electric Power, Ph.D. Dissertation, Department of Economics, Harvard University, Cambridge, Mass., 1967.

264. Masse, P. and F. Bessiere, "Long-Term Programming of Electrical Investments", Marginal Cost Pricing in Practice, Prentice-Hall, Englewood Cliff, New Jersy, 1964.

265. Masse, P. and R. Gibrat, "Application of Linear Programming to Investments in the Electrical Power Industry", Management Science, Vol. 3, 1957, p. 149.

266. Dantzig, G.B., Linear Programming and Extensions, Princeton University Press, Princeton, New Jersey, 1963.

267. Bessiere, F., "The Investment 85 Model of Electricite de France", Management Science, 17, 1970? p. B.192, B. 211, 1970.

268. Lack, G.N., "Optimisation Studies with Application to Planning in the Electric Power Industry and Optimal Control Theory", IEES, Report CCS-5, Stanford University, Stanford, California, August, 1968.

269. Chen, K and K. Linvill, "Some Special Problems of Electric Power Expansion in Developing Countries", Symposium on the Possibilities of O.R. in Developing Countries, Paris, June 1963.

270. Sautter, E.A., "Studies in the Long Range Planning of Inter tie Between Electric Power Systems", Report EEP 11, IEES, Stanford, California, 1964.

271. Narasimhan, R., N. Seshagiri et. al., "A Mix Optimisation Model and its Application to, the Northern Grid", Proceedings of the Seminar on Nuclear Power Bombay, Jan. 17-18, 1970, p. 92.

272. Gosai, P.S., "Regional Power Planning - A Linear Programming Approach", An M.Tech. Thesis, Submitted to I.I.T. Kanpur, India, July 1973.

273. Tikaria, O.P., "Capacity Planning in an Integrated Power System", An M.Tech. Thesis, Submitted to I.I.T. Kanpur, October, 1973.

274. Dale, K.M., "Dynamic Programming Approach to the Selection and Timing of Generation-Plant Additions", Proceedings of I.E.E. (G.B.), Vol. 113, No. 5, May 1966, p. 803.

275. Okada, T., and Y. Kawai, "Expansion Planning of Power System Taking into Account, The Step-wise Cost Characteristics", Electrical Engineering in Japan, Vol. 90, No. 4, 2970, p. 122.

276. Marglin, S.A., "Approaches to Dynamic Investment Planning, North Holland Pub. Co., Amsterdam.

277. Nelson, J.R. (ed.), Marginal Cost Pricing in Practice, Prentice Hall Publications, N.J. 1971

278. Dary, C.W., Operational Economics of Electricity Utilities, Columbia Univ. Press, N.Y., 1968

279. Cazallet, E.G., "Decomposition of Complex Decision Problems with Application to Electric Power Systems Planning," Ph.D. Dissertation, Stanford University, Stanford, California, May. 1970.

280. Stanford Research Institute "Decision Analysis of Nuclear Plants in Electric System Expansion", Report prepared for the Commision Federal de Electricidad, Mexico City, Menlo Park, December 1968.

281. Baldwin, C.J., et. al., "A Model for Transmission Planning by Logic," AIEE Trans (PAS), Vol. 46, Feb. 1960, p. 1638.

282. Baldwin, C.J., et. al., "The Effect of Unit Size Reliability and Systems Service Quality in Planning Generation Expansions", AIEE Trans., (PAS), Vol. 52, Feb. 1961, p. 1042.

283. Benders, J.F., "Partition Procedures for Solving Mixed Variable Programme Problems", Numerische Mathematik, Vol. 4, 1962, p. 238.

284. Geoffrion, A.M., "Generalised Benders Decomposition", Working Paper No. 159, Western Management Science Institute, UCLA, September, 1970.

285. Geoffrion, A.M., "Implicit Enumeration Using an Imbeded L.P. - Memorandum, RM-5406, The Rand Corporation, September, 1962.

286. Wilson, R., "Programming Variable Factors", Management Science, Vol. 13, No. 1, Sept. 1966, p. 144.

287. Holcombe, B.D., "An Implementation of a Zero One Programming Algorithm," Union Carbide Corporation, Nuclear Division, Oakridge, Tennesse, Report No. CIC-3, Sept. 6, 1968.

288. Land, A.H. and Doig, A, "An Automatic Method of Solving Discrete Programming Problems", Econometrica, Vol. 28, 1960, p. 497.

289. Beiter, S. and G.R. Sherman, "Discrete Optimisation, Journal of the Society Ind. App. Math., Vol. 13, No. 3, 1965, p. 864.

290. Geoffrion, A.M., "Element of Large Scale Mathematical Programming", Working Paper No. 144, Western Management Science Institute, UCLA, July 1970.

291. Balas, E., "An Additive Algorithm for Solving Linear Programming with Zero One Variables", Operations Research, Vol. 13, No. 4, July August 1965, p. 517.

292. Manne, A.S., "Waiting for the Breeder", Review of Economics Studies Symposium, 1974, p. 47.

APPENDIX - A

I. SOURCES OF DATA FOR FORECASTING ELECTRICITY DEMAND FOR INDIA FOR THE PERIOD 1976 - 2000

1. Planning Commission, Government of India, New Delhi, India.

2. Annual Survey of Public Utilities (Publication of the CWPC, Government of India, New Delhi).

3. Central Statistical Organisation Data Files, New Delhi, Government of India, New Delhi.

4. Load Survey Reports for all the States of India (CWPC Publications).

II. SOURCES OF DATA FOR ANALYSING INDUSTRIAL ELECTRICITY DEMAND

1. Annual Survey of Industries 1954 - 1972
Central Statistical Organisation
Government of India
New Delhi

2. Census of Manufacturing in Industries
Ministry of Industries
Government of India
New Delhi

III. SOURCES OF DATA FOR CAPACITY EXPANSION OF ELECTRIC POWER SYSTEMS

1. Publications of the C.F.E.
Commission Federal de Electricidad
Mexico City, Menlo Park - December 1968

2. Survey of Electricity Demand for the Sonora - Siraloa Region, Publications of CFE
Hermosllo, Mexico, 1971